High School Math Enrichment

Wayne-Westland Community Schools

Hopefully everyone is staying healthy and safe during this time. Please use these activities to enrich your understanding of Mathematics during this time.

The lessons can be done in any order. They are just titled so you can easily find them.

LESSON 3.4

Study Guide

For use with pages 154–160

GOAL **Solve equations with variables on both sides.**

Vocabulary

An equation that is true for all values of the variable is an **identity.**

EXAMPLE 1

Solve an equation with variables on both sides

Solve $13 - 6x = 3x - 14$.

Solution

$13 - 6x = 3x - 14$	Write original equation.
$13 - 6x + 6x = 3x - 14 + 6x$	Add $6x$ to each side.
$13 = 9x - 14$	Simplify.
$27 = 9x$	Add 14 to each side.
$3 = x$	Divide each side by 9.

The solution is 3. Check by substituting 3 for x in the original equation.

CHECK $13 - 6x = 3x - 14$	Write original equation.
$13 - 6(3) = 3(3) - 14$	Substitute 3 for x.
$-5 = 3(3) - 14$	Simplify left side.
$-5 = -5$ ✓	Simplify right side. Solution checks.

Exercises for Example 1

Solve the equation. Check your solution.

1. $9a = 7a - 8$ **2.** $17 - 8b = 3b - 5$ **3.** $-5c + 6 = 9 - 4c$

EXAMPLE 2

Solve an equation with grouping symbols

Solve $4x - 7 = \frac{1}{3}(9x - 15)$.

Solution

$4x - 7 = \frac{1}{3}(9x - 15)$	Write original equation.
$4x - 7 = 3x - 5$	Distributive property
$x - 7 = -5$	Subtract $3x$ from each side.
$x = 2$	Add 7 to each side.

The solution is 2.

LESSON 3.4 **Study Guide** *continued*
For use with pages 154–160

Exercises for Example 2

Solve the equation. Check your solution.

4. $2m - 7 = 3(m + 8)$

5. $\frac{1}{5}(15n + 5) = 8n - 9$

6. $7p - 3 = \frac{3}{4}(8p - 12)$

EXAMPLE 3 Identify the number of solutions of an equation

Solve the equation, if possible.

a. $4(3x - 2) = 2(6x + 1)$ **b.** $4(4x - 5) = 2(8x - 10)$

Solution

a. $4(3x - 2) = 2(6x + 1)$ Write original equation.

$12x - 8 = 12x + 2$ Distributive property

$12x = 12x + 10$ Add 8 to each side.

The equation $12x = 12x + 10$ is not true because the number $12x$ cannot be equal to 10 more than itself. So, the equation has no solution. This can be demonstrated by continuing to solve the equation.

$12x - 12x = 12x + 10 - 12x$ Subtract $12x$ from each side.

$0 = 10$ Simplify.

The statement $0 = 10$ is not true, so the equation has no solution.

b. $4(4x - 5) = 2(8x - 10)$ Write original equation.

$16x - 20 = 16x - 20$ Distributive property

Notice that the statement $16x - 20 = 16x - 20$ is true for all values of x. So, the equation is an identity.

Exercises for Example 3

Solve the equation, if possible.

7. $11x + 7 = 10x - 8$

8. $5(3x - 2) = 3(5x - 1)$

9. $\frac{1}{2}(6x + 18) = 3(x + 3)$

LESSON 1.3

Study Guide

For use with pages 18–25

GOAL **Solve linear equations.**

Vocabulary

An **equation** is a statement that two expressions are equal.

A **linear equation** in one variable is an equation that can be written in the form $ax + b = 0$ where a and b are constants and $a \neq 0$.

A number is a **solution** of an equation if substituting the number for the variable results in a true statement.

Two equations are **equivalent equations** if they have the same solution(s).

EXAMPLE 1 Solve an equation with a variable on one side

Solve $6x - 8 = 10$.

Solution

$6x - 8 = 10$	Write original equation.
$6x = 18$	Add 8 to each side.
$x = 3$	Divide each side by 6.

EXAMPLE 2 Solve an equation with a variable on both sides

Solve $8z + 7 = -2z - 3$.

Solution

$8z + 7 = -2z - 3$	Write original equation.
$10z + 7 = -3$	Add $2z$ to each side.
$10z = -10$	Subtract 7 from each side.
$z = -1$	Divide each side by 10.

Exercises for Examples 1 and 2

Solve the equation. Check your solution.

1. $14x = 7$ **2.** $3n + 2 = 14$ **3.** $-6t - 5 = 13$

4. $11q - 4 = 6q - 9$ **5.** $5a - 1 = 2a + 11$ **6.** $-2m + 3 = 7m - 6$

LESSON **1.3**

Study Guide *continued*

For use with pages 18–25

EXAMPLE 3 Solve an equation using the distributive property

Solve $2(3x + 1) = -3(x - 2)$.

Solution

$2(3x + 1) = -3(x - 2)$	Write original equation.
$6x + 2 = -3x + 6$	Distributive property
$9x + 2 = 6$	Add $3x$ to each side.
$9x = 4$	Subtract 2 from each side.
$x = \frac{4}{9}$	Divide each side by 9.

Exercises for Example 3

7. Solve $4(2x - 1) = 3(x + 2)$.

8. Solve $5(x + 3) = -(x - 3)$.

EXAMPLE 4 Solve a work problem

It takes you 3 hours to mow a lawn and it takes your sister 2 hours to mow a lawn. How long does it take the two of you to mow 5 lawns if you work together?

Solution

Solve the equation $\frac{1}{3}t + \frac{1}{2}t = 5$ for t.

$\frac{1}{3}t + \frac{1}{2}t = 5$	Write equation.
$6\left(\frac{1}{3}t + \frac{1}{2}t\right) = 6(5)$	Multiply each side by the LCD, 6.
$2t + 3t = 30$	Distributive property
$5t = 30$	Combine like terms.
$t = 6$	Divide each side by 5.

It will take 6 hours to mow 5 lawns if you work together.

Exercise for Example 4

9. Rework Example 4 to find how long it takes the two of you to mow 12 lawns if you work together.

LESSON 1.4

Study Guide

For use with pages 26–32

GOAL **Rewrite and evaluate formulas and equations.**

Vocabulary

A **formula** is an equation that relates two or more quantities, usually represented by variables.

To **solve for a variable** means to rewrite an equation as an equivalent equation in which the variable is on one side and does not appear on the other side.

EXAMPLE 1

Rewrite a formula with two variables

Solve the formula $F = \frac{9}{5}C + 32$ for C.

$F = \frac{9}{5}C + 32$	Write temperature formula.
$5F = 9C + 160$	Multiply each side by 5.
$5F - 160 = 9C$	Subtract 160 from each side.
$\frac{5F - 160}{9} = C$	Divide each side by 9.

EXAMPLE 2

Rewrite a formula with three variables

Solve the formula $A = \ell w$ for ℓ. Then find ℓ when $A = 5$ square centimeters and $w = 2$ centimeters.

STEP 1 **Solve** the formula for ℓ .

$A = \ell w$	Write area formula.
$\frac{A}{w} = \frac{\ell w}{w}$	Divide each side by w.
$\frac{A}{w} = \ell$	Simplify.

STEP 2 **Substitute** the given values into the rewritten formula.

$\ell = \frac{5}{2} = 2.5$	Substitute 5 for A and 2 for w.

The length of the rectangle is 2.5 centimeters.

Exercises for Examples 1 and 2

1. Solve the formula $C = 2\pi r$ for r. Then find the radius of a circle with a circumference of 88 inches.

2. Solve the formula $P = 2\ell + 2w$ for w. Then find the width of a rectangle with a length of 11.5 centimeters and a perimeter of 92 centimeters.

LESSON 1.4 **Study Guide** *continued*
For use with pages 26–32

EXAMPLE 3 Rewrite a nonlinear equation

Solve the equation $xy - y = 9$ for y. Then find the value of y for $x = 4$.

STEP 1 **Solve** the equation for y.

$xy - y = 9$	Write original equation.
$y(x - 1) = 9$	Distributive property
$y = \frac{9}{x - 1}$	Divide each side by $x - 1$.

STEP 2 **Substitute** the given value into the rewritten equation.

$y = \frac{9}{4 - 1} = 3$	Substitute 4 for x and simplify.

Exercises for Example 3

Solve the equation for y. Find the value of y for the given value of x.

3. $xy + y = 8; x = 3$ **4.** $2xy + y = 6; x = 1$ **5.** $4xy - y = 7; x = 2$

EXAMPLE 4 Solve a multi-step problem

You buy x shirts for $8 each and y hats for $4 each. Write an equation that represents your total purchases T (in dollars) and solve the equation for y. Evaluate the equation for $T = 28$ and $x = 2$.

STEP 1 **Write** a verbal model. Then write an equation.

Total purchases	=	Prices of shirts	•	Number of shirts	+	Price of hats	•	Number of hats

An equation is $T = 8x + 4y$.

STEP 2 **Solve** the equation for y.

$T = 8x + 4y$	Write equation.
$T - 8x = 4y$	Subtract $8x$ from each side.
$\frac{T - 8x}{4} = y$	Divide each side by 4.

STEP 3 **Calculate** y. If $T = 28$ and $x = 2$, then $y = \frac{28 - 8(2)}{4} = 3$.

If 2 shirts are purchased, then 3 hats are purchased.

Exercise for Example 4

6. In Example 4, how many hats can you buy if you buy 1 shirt and spend $16?

1

If $5x + 6 = 10$, what is the value of $10x + 3$?

A) 4
B) 9
C) 11
D) 20

3

$$\ell = 24 + 3.5m$$

One end of a spring is attached to a ceiling. When an object of mass m kilograms is attached to the other end of the spring, the spring stretches to a length of ℓ centimeters as shown in the equation above. What is m when ℓ is 73 ?

A) 14
B) 27.7
C) 73
D) 279.5

$$I = \frac{P}{4\pi r^2}$$

At a large distance r from a radio antenna, the intensity of the radio signal I is related to the power of the signal P by the formula above.

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Which of the following expresses the square of the distance from the radio antenna in terms of the intensity of the radio signal and the power of the signal?

A) $r^2 = \frac{IP}{4\pi}$
B) $r^2 = \frac{P}{4\pi I}$
C) $r^2 = \frac{4\pi I}{P}$
D) $r^2 = \frac{I}{4\pi P}$

2

If $3r = 18$, what is the value of $6r + 3$?

A) 6
B) 27
C) 36
D) 39

5

If $\frac{5}{x} = \frac{15}{x + 20}$, what is the value of $\frac{x}{5}$?

A) 10
B) 5
C) 2
D) $\frac{1}{2}$

7

If $\frac{3}{5}w = \frac{4}{3}$, what is the value of w ?

A) $\frac{9}{20}$
B) $\frac{4}{5}$
C) $\frac{5}{4}$
D) $\frac{20}{9}$

17

$$2(p+1)+8(p-1)=5p$$

What value of p is the solution of the equation above?

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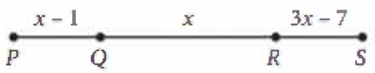


Note: Figure not drawn to scale.

On $\overline{PS}$ above, $PQ = RS$. What is the length of $\overline{PS}$?

7

A bricklayer uses the formula $n = 7\ell h$ to estimate the number of bricks, n, needed to build a wall that is ℓ feet long and h feet high. Which of the following correctly expresses ℓ in terms of n and h ?

A) $\ell = \dfrac{7}{nh}$

B) $\ell = \dfrac{h}{7n}$

C) $\ell = \dfrac{n}{7h}$

D) $\ell = \dfrac{n}{7+h}$

4

If $3(c+d)=5$, what is the value of $c+d$?

A) $\dfrac{3}{5}$

B) $\dfrac{5}{3}$

C) 3

D) 5

6

If $x = \dfrac{2}{3}y$ and $y = 18$, what is the value of $2x - 3$?

A) 21

B) 15

C) 12

D) 10

32

$$2(5x-20)-(15+8x)=7$$

What value of x satisfies the equation above?

2

Which of the following is equivalent to $3(x+5)-6$?

A) $3x-3$

B) $3x-1$

C) $3x+9$

D) $15x-6$

1

$$3x+x+x+x-3-2=7+x+x$$

In the equation above, what is the value of x ?

A) $-\frac{5}{7}$

B) 1

C) $\frac{12}{7}$

D) 3

3

The formula below is often used by project managers to compute E, the estimated time to complete a job, where O is the shortest completion time, P is the longest completion time, and M is the most likely completion time.

$$E=\frac{O+4M+P}{6}$$

Which of the following correctly gives P in terms of E, O, and M ?

A) $P=6E-O-4M$

B) $P=-6E+O+4M$

C) $P=\frac{O+4M+E}{6}$

D) $P=\frac{O+4M-E}{6}$

12

If $\frac{2a}{b}=\frac{1}{2}$, what is the value of $\frac{b}{a}$?

A) $\frac{1}{8}$

B) $\frac{1}{4}$

C) 2

D) 4

7

If $\frac{8}{x}=160$, what is the value of x ?

A) 1,280

B) 80

C) 20

D) 0.05

1

What value of x satisfies the equation $3x+3=27$?

A) 3

B) 8

C) 10

D) 27

3

If $\frac{2n}{5} = 10$, what is the value of $2n - 1$?

A) 24

B) 49

C) 50

D) 99

9

$$(-4x + 5) - (6x + 7) = 0$$

What is the solution to the equation above?

A) $x = 6$

B) $x = 1$

C) $x = -0.2$

D) $x = -1.2$

17

$$\frac{2}{3}t = \frac{5}{2}$$

What value of t is the solution of the equation above?

32

$$x - \frac{1}{2}a = 0$$

If $x = 1$ in the equation above, what is the value of a ?

12

$$(x^2y^3)^{\frac{1}{2}}(x^2y^3)^{\frac{1}{3}} = x^{\frac{a}{3}}y^{\frac{a}{2}}$$

If the equation above, where a is a constant, is true for all positive values of x and y, what is the value of a ?

A) 2

B) 3

C) 5

D) 6

16

If x is not equal to zero, what is the value of $\frac{4(3x)^2}{(2x)^2}$?

Lesson 4: Write Equations of Lines

Point Slope Form

Point Slope: $y - y_1 = m(x - x_1)$

What is this really saying? Why does this work?

Well lets look at it piece by piece

$(x - x_1)$

Here we are subtracting our starting x value (x_1) from our current value.
This tells us how <u>much x has changed</u> from the start.

$m(x - x_1)$

m is slope, which is $\frac{\text{change in y}}{\text{change in x}}$ and $(x - x_1)$ is the change in x

so $\frac{\text{change in y}}{\text{change in x}}$ * change in x

change in x divides to 1, so this must just be change in y

$y - y_1$

Here we are subtracting our starting y value (y_1) from our current value.
This tells us how <u>much y has changed</u> from the start.

$y - y_1 = m(x - x_1)$

so this is really just saying that the change in y is the change in y

This is good. Equations should have the same result on both sides.

An example: Write the equation of a line that passes through (-4, 6) and has a slope of $\frac{2}{3}$

$y - 6 = \frac{2}{3}(x - -4)$ just plugging the values in

$y - 6 = \frac{2}{3}(x + 4)$

Another example: Write the equation of the line that passes through (5, 10) and (9, 4)

Here we have two points. We need a slope.

Calculating the slope $m = \frac{4 - 10}{9 - 5} = \frac{-6}{4}$

simplifying the fraction $m = \frac{-3}{2}$

Now we have the slope and two points.
We only need one.
Pick your favorite.

(5, 10)

$y - 10 = \frac{-3}{2}(x - 5)$

(9, 4)

$y - 4 = \frac{-3}{2}(x - 9)$

Either is correct (this is one reason that students do not like this form - there is not one right answer)

There are multiple equations because there are an infinite number of points along a line.

Slope Intercept Form

Slope Intercept: $y = mx + b$

This is really just the same thing as point slope form.

We just pick a very specific point (the y intercept) and solve for y.

In the grand scheme of lines, the probability that if we pick a random point on a line and it is the y intercept is zero - that is why point slope is more general (and better)

Point Slope: $y - y_1 = m(x - x_1)$

Let our slope still be *m*, but lets have our point be the y intercept expressed as (0, *b*)

plugging in these values	$y - b = m(x - 0)$
simplifying	$y - b = mx$
adding *b* to both sides	$y = mx + b$

So really, slope intercept form is just a specific example of point slope.

An example: A line has slope of 5 and y intercept of 4

$$y = 5x + 4$$

It is just a matter of substituting things

A harder example: Write the equation of a line *in slope intercept form* if the line passes through (4, 6) and has slope 5.

Here, we don't know the y intercept.
We still use the same equation, but now we plug in what we know and solve for *b*

$$y = mx + b$$

$$6 = 5 * 4 + b$$

$$6 = 20 + b$$

$$-14 = b$$

we then need to substitute b into our original equation with the slope

y = 5x - 14

we could use point slope form as well, just making sure we solve for y

$$y - 6 = 5(x - 4)$$

$$y - 6 = 5x - 20$$

$$y = 5x - 14$$

LESSON 2.4
Study Guide
For use with pages 98–104

GOAL **Write linear equations.**

Vocabulary

The **point-slope form** of the equation of a line is given by $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

EXAMPLE 1 Write an equation given the slope and a point

a. Write an equation with a slope of −2 and a y-intercept of 3.

b. Write an equation with a slope of 4 that passes through the point (−1, 7).

Solution

a. $y = mx + b$ — Use slope-intercept form.
$y = -2x + 3$ — Substitute −2 for m and 3 for b.

b. $y - y_1 = m(x - x_1)$ — Use point-slope form.
$y - 7 = 4(x + 1)$ — Substitute for m, x_1, and y_1.
$y - 7 = 4x + 4$ — Distributive property
$y = 4x + 11$ — Write in slope-intercept form.

EXAMPLE 2 Write equations of parallel or perpendicular lines

Write an equation of the line that passes through (0, 1) and is (a) parallel to, and (b) perpendicular to, the line $y = 2x + 7$.

Solution

a. The given line has a slope of $m_1 = 2$. A line parallel to the given line has a slope of $m_2 = 2$. Use the point-slope form with $(x_1, y_1) = (0, 1)$ and $m_2 = 2$ to write an equation of the line.

$y - y_1 = m(x - x_1)$ — Use point-slope form.
$y - 1 = 2(x - 0)$ — Substitute for m, x_1, and y_1.
$y - 1 = 2x - 2(0)$ — Distributive property
$y = 2x + 1$ — Write in slope-intercept form.

b. A line perpendicular to a line with slope $m_1 = 2$ must have a slope of $m_2 = -\frac{1}{m_1} = -\frac{1}{2}$. Use the point-slope form with $(x_1, y_1) = (0, 1)$.

$y - y_1 = m(x - x_1)$ — Use point-slope form.
$y - 1 = -\frac{1}{2}(x - 0)$ — Substitute for m, x_1, and y_1.
$y - 1 = -\frac{1}{2}x - \left(-\frac{1}{2}\right)(0)$ — Distributive property
$y = -\frac{1}{2}x + 1$ — Write in slope-intercept form.

LESSON 2.4

Study Guide *continued*

For use with pages 98–104

Exercises for Examples 1 and 2

Write an equation of the line with the given conditions.

1. With a slope of 4 and a y-intercept of -1
2. With a slope of -5 that passes through the point $(3, -2)$
3. That passes through $(2, 3)$ and parallel to $y = -x + 3$
4. That passes through $(0, 1)$ and perpendicular to $y = 2x + 7$

EXAMPLE 3 Write an equation given two points

Write an equation of the line that passes through (2, 8) and (4, 14).

Find the slope of the line through $(x_1, y_1) = (2, 8)$ and $(x_2, y_2) = (4, 14)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 8}{4 - 2} = \frac{6}{2} = 3$$

Use the point-slope form with the point (2, 8) to write an equation.

$y - y_1 = m(x - x_1)$	Use point-slope form.
$y - 8 = 3(x - 2)$	Substitute for m, x_1, and y_1.
$y - 8 = 3x - 6$	Distributive property
$y = 3x + 2$	Write in slope-intercept form.

Exercises for Example 3

Write an equation of the line that passes through the given points.

5. $(1, -1), (4, 2)$
6. $(-2, 4), (3, -1)$
7. $(-3, -1), (0, 1)$

EXAMPLE 4 Write a model using standard form

You have $16 to spend on pens and binders. Binders cost $2 each and pens costs $3 each. Write an equation that models this situation.

Write a verbal model. Then write an equation.

Price of binders	•	Number of binders	+	Price of pens	•	Number of pens	=	Your budget
2	•	x	+	3	•	y	=	16

An equation for this situation is $2x + 3y = 16$.

Exercise for Example 4

8. You have $70 to spend on sweaters and jeans. Sweaters cost $15 each and jeans cost $20 each. Write an equation that models this situation.

LESSON 3.2

Study Guide

For use with pages 160–167

GOAL **Solve linear systems algebraically.**

Vocabulary

To use the **substitution method,** Step 1 is to *solve* one of the equations for one of its variables. Step 2 is to *substitute* the expression from Step 1 into the other equation and solve for the other variable. Step 3 is to *substitute* the value from Step 2 into the revised equation from Step 1 and solve.

To use the **elimination method,** Step 1 is to *multiply* one or both of the equations by a constant to obtain coefficients that differ only in sign for one of the variables. Step 2 is to *add* the revised equations from Step 1 and solve for the remaining variable. Step 3 is to *substitute* the value obtained in Step 2 into either of the original equations and solve for the other variable.

EXAMPLE 1 Use the substitution method

Solve the system using the substitution method.

$6x + 3y = 12$ **Equation 1**

$3x + y = 5$ **Equation 2**

Solution

STEP 1 **Solve** Equation 2 for y.

$y = 5 - 3x$

STEP 2 **Substitute** the expression for y into Equation 1 and solve for x.

$6x + 3(5 - 3x) = 12$ Substitute $5 - 3x$ for y.

$x = 1$ Solve for x.

STEP 3 **Substitute** the value of x into Equation 2 and solve for y.

$3(1) + y = 5$ Substitute 1 for x.

$y = 2$ Solve for y.

The solution is (1, 2).

Exercises for Example 1

Solve the system using the substitution method.

1. $2x + y = 4$
$3x - 5y = 6$

2. $3x + 6y = 3$
$x - 2y = 5$

3. $2x - y = 6$
$-3x + 2y = -8$

LESSON 3.2 **Study Guide** *continued*
For use with pages 160–167

EXAMPLE 2 Use the elimination method

Solve the system using the elimination method.

$x + 5y = 13$ **Equation 1**
$-4x - 7y = -13$ **Equation 2**

STEP 1 **Multiply** Equation 1 by 4 so that the coefficients of x differ only in sign.

$x + 5y = 13$ $\xrightarrow{\times 4}$ $4x + 20y = 52$
$-4x - 7y = -13$ $\quad$ $-4x - 7y = -13$

STEP 2 **Add** the revised equations and solve for y. $13y = 39$
$y = 3$

STEP 3 **Substitute** the value of y into Equation 1 and solve for x.

$x + 5(3) = 13$ Substitute 3 for y in Equation 1.
$x = -2$ Solve for x.

The solution is $(-2, 3)$.

EXAMPLE 3 Solve linear systems with many or no solutions

a. Solve: $9x - 3y = 6$
$3x - y = 2$

b. Solve: $x - 2y = 5$
$4x - 8y = 3$

a. $y = 3x - 2$ Solve Equation 2 for y.
$9x - 3(3x - 2) = 6$ Substitute $3x - 2$ for y.
$6 = 6$ Simplify.

Because the equation $6 = 6$ is always true, there are infinitely many solutions.

b. $-4x + 8y = -20$ Multiply Equation 1 by -4.
$4x - 8y = 3$ Add Equation 2.
$0 = -17$

Because the equation $0 = -17$ is never true, there is no solution.

Exercises for Examples 2 and 3

Solve the system using the elimination method.

4. $7x + 2y = -5$
$3x - 4y = -7$

5. $2x - 3y = 3$
$4x - 5y = 9$

6. $5x - 6y = 4$
$2x + 3y = 7$

Solve the linear system by any algebraic method.

7. $3x - y = 5$
$6x - 2y = 10$

8. $x - 2y = 5$
$4x - 8y = -3$

9. $x + 2y = 7$
$-3x - 6y = -7$

2

$$x + y = 0$$
$$3x - 2y = 10$$

Which of the following ordered pairs (x, y) satisfies the system of equations above?

A) $(3, -2)$

B) $(2, -2)$

C) $(-2, 2)$

D) $(-2, -2)$

9

$$kx - 3y = 4$$
$$4x - 5y = 7$$

In the system of equations above, k is a constant and x and y are variables. For what value of k will the system of equations have no solution?

A) $\frac{12}{5}$

B) $\frac{16}{7}$

C) $-\frac{16}{7}$

D) $-\frac{12}{5}$

6

$$2x - 3y = -14$$
$$3x - 2y = -6$$

If (x, y) is a solution to the system of equations above, what is the value of $x - y$?

A) -20

B) -8

C) -4

D) 8

9

$$y = x^2$$
$$2y + 6 = 2(x + 3)$$

If (x, y) is a solution of the system of equations above and $x > 0$, what is the value of xy ?

A) 1

B) 2

C) 3

D) 9

11

$$7x + 3y = 8$$
$$6x - 3y = 5$$

For the solution (x, y) to the system of equations above, what is the value of $x - y$?

A) $-\frac{4}{3}$

B) $\frac{2}{3}$

C) $\frac{4}{3}$

D) $\frac{22}{3}$

14

$$y = x^2 + 3x - 7$$
$$y - 5x + 8 = 0$$

How many solutions are there to the system of equations above?

A) There are exactly 4 solutions.

B) There are exactly 2 solutions.

C) There is exactly 1 solution.

D) There are no solutions.

3

$$x = y - 3$$
$$\frac{x}{2} + 2y = 6$$

Which ordered pair (x, y) satisfies the system of equations shown above?

A) $(-3, 0)$

B) $(0, 3)$

C) $(6, -3)$

D) $(36, -6)$

15

$$g(x) = 2x - 1$$
$$h(x) = 1 - g(x)$$

The functions g and h are defined above. What is the value of $h(0)$?

A) -2

B) 0

C) 1

D) 2

LESSON 3.4

Study Guide

For use with pages 177–185

GOAL **Solve systems of equations in three variables.**

Vocabulary

A **linear equation in three variables** x, y, and z is an equation of the form $ax + by + cz = d$ where a, b, and c are not all zero.

An example of a **system of three linear equations** in three variables:

$x + 2y + z = 3$ Equation 1

$2x + y + z = 4$ Equation 2

$x - y - z = 2$ Equation 3

A **solution of a system with three variables** is an **ordered triple** (x, y, z) whose coordinates make each equation true.

EXAMPLE 1

Use the elimination method

Solve the system.

$2x + 3y - z = 13$ **Equation 1**

$3x + y - 3z = 11$ **Equation 2**

$x - y + z = 3$ **Equation 3**

STEP 1 **Rewrite** the system as a linear system in two variables.

$$\begin{array}{r} 2x + 3y - z = 13 \\ 3x - 3y + 3z = 9 \\ \hline 5x + 2z = 22 \end{array}$$

Add 3 times the third equation to the first equation. New Equation 1

$$\begin{array}{r} 3x + y - 3z = 11 \\ x - y + z = 3 \\ \hline 4x - 2z = 14 \end{array}$$

Add the second and third equations. New Equation 2

STEP 2 **Solve** the new linear system for both of its variables.

$$\begin{array}{r} 5x + 2z = 22 \\ 4x - 2z = 14 \\ \hline 9x = 36 \end{array}$$

Add new Equation 1 and new Equation 2.

$x = 4$ Solve for x.

$z = 1$ Substitute into new Equation 1 or 2 to find z.

STEP 3 **Substitute** $x = 4$ and $z = 1$ into an original equation and solve for y.

$x - y + z = 3$ Write original Equation 3.

$4 - y + 1 = 3$ Substitute 4 for x and 1 for z.

$y = 2$ Solve for y.

The solution is $x = 4$, $y = 2$, and $z = 1$ or the ordered triple $(4, 2, 1)$.

EXAMPLE 2 Solve a three-variable system with no solution

Solve the system.

$$\begin{aligned} 2x - 2y + 2z &= 9 && \textbf{Equation 1} \\ x - y + z &= 5 && \textbf{Equation 2} \\ 3x + y + 2z &= 4 && \textbf{Equation 3} \end{aligned}$$

When you multiply the second equation by -2 and add the result to the first equation, you obtain a false equation.

$$\begin{aligned} 2x - 2y + 2z &= 9 \\ -2x + 2y - 2z &= -10 \\ 0 &= -1 && \text{New Equation 1} \end{aligned}$$

Because you obtain a false equation, the original system has no solution.

EXAMPLE 3 Solve a three-variable system with many solutions

Solve the system.

$$\begin{aligned} x + y + z &= 3 && \textbf{Equation 1} \\ x + y - z &= 3 && \textbf{Equation 2} \\ 3x + 3y + z &= 9 && \textbf{Equation 3} \end{aligned}$$

STEP 1 **Rewrite** the system as a linear system in two variables.

$$\begin{aligned} x + y + z &= 3 && \text{Add the first equation} \\ x + y - z &= 3 && \text{to the second.} \\ \hline 2x + 2y &= 6 && \text{New Equation 1} \end{aligned}$$

$$\begin{aligned} x + y - z &= 3 && \text{Add the second equation} \\ 3x + 3y + z &= 9 && \text{to the third.} \\ \hline 4x + 4y &= 12 && \text{New Equation 2} \end{aligned}$$

STEP 2 **Solve** the new linear system for both of its variables.

$$\begin{aligned} -4x - 4y &= -12 && \text{Add } -2 \text{ times new Equation 1} \\ 4x + 4y &= 12 && \text{and new Equation 2.} \\ \hline 0 &= 0 \end{aligned}$$

Because you obtain the identity $0 = 0$, the system has infinitely many solutions.

STEP 3 **Describe** the solution. Divide new Equation 1 by 2 to get $x + y = 3$, or $y = -x + 3$. Substituting this into the original equation produces $z = 0$. Any ordered triple of the form $(x, -x + 3, 0)$ is a solution of the system.

Exercises for Examples 1, 2, and 3

Solve the system.

1. $2x + y + z = 5$
$x - y + 2z = 4$
$x + y + z = 4$

2. $x + 2y + z = 7$
$x - y + z = 4$
$3x + 6y + 3z = 9$

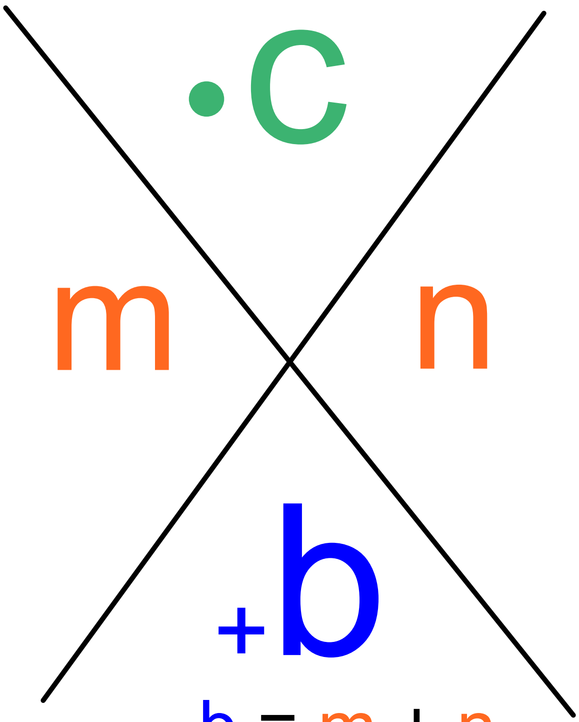
3. $x + y + z = 2$
$x + y - z = 2$
$2x + 2y + z = 4$

Factoring

In general:

$(x \pm m)(x \pm n) = x^2 \pm bx \pm c$

$c = m \cdot n$



$b = m + n$

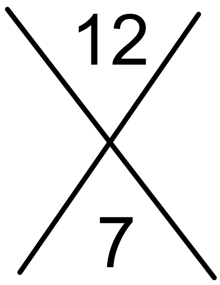
$(x \pm m)(x \pm n) = x^2 \pm bx \pm c$

Example:

$x^2 \pm bx \pm c$

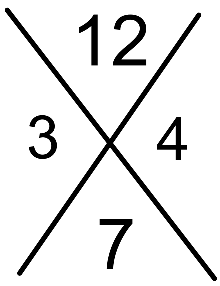
Factor $x^2 + 7x + 12$

I am looking for two numbers that multiply to 12 but also add to 7



1*12	-1*-12
2*6	-2*-6
3*4	-3*-4

1+12 =13	-1+-12=-13
2+6 = 8	-2+-6 = -8
3+4 =7	-3+-4 = -7



$(x + 3)(x+4)$

Factored

$x^2 + 7x + 12 = (x + 3)(x+4)$

Lesson 8: Factoring with Leading Coefficient of 1

Factor the quadratics below

You can draw an X if you find it helpful but your answers should be in factored form

(x ± m)(x ± n)

1) $x^2 + 7x + 12$

6) $x^2 - 6x + 9$

2) $x^2 - 4$

rewrite these problems with b=0

ex: $x^2 + 0x - 4$

7) $x^2 - 8x - 9$

3) $x^2 - 25$

8) $h^2 + 6h + 8$

4) $g^2 + 10g + 21$

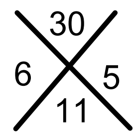
9) $x^2 + 5x - 50$

5) $x^2 + 10x + 25$

10) $x^2 + 12x + 32$

Factoring using the bottoms up method

Steps for Bottom's Up:	$ax^2 \pm bx \pm c$
1) Multiply the first coefficient (*a*) and the last number to convert to $x^2 \pm bx \pm c$	$2x^2 + 11x + 15$ 2*15= 30 Step 1: $x^2 + 11x + 30$
2) Use the organizational X and factor as we did previously	Step 2: 6*5 = 30 6+5 = 11
3) BEFORE you put anything into parentheses, divide both of your factors by the original leading coefficient (*a*) 4) *Simplify* each fraction	Step 3&4: $\frac{6}{2} = \frac{3}{1}$ $\frac{5}{2}$ simpified
5) Put into parentheses and bring the bottom up by putting it as the coefficient of the variable.	Step 5: $(x + \frac{3}{1})\ (x + \frac{5}{2})$ Factored Form: (x + 3)(2x+5)



Lesson 9: Factoring with Leading Coefficient not 1

Factoring Trinomials (a > 1)

Factor each completely.

1) $3p^2 - 2p - 5$

2) $2n^2 + 3n - 9$

3) $3n^2 - 8n + 4$

4) $5n^2 + 19n + 12$

5) $2v^2 + 11v + 5$

6) $2n^2 + 5n + 2$

7) $7a^2 + 53a + 28$

8) $9k^2 + 66k + 21$

Lesson 9: Factoring with Leading Coefficient not 1

9) $15n^2 - 27n - 6$

10) $5x^2 - 18x + 9$

11) $4n^2 - 15n - 25$

12) $4x^2 - 35x + 49$

13) $4n^2 - 17n + 4$

14) $6x^2 + 7x - 49$

15) $6x^2 + 37x + 6$

16) $-6a^2 - 25a - 25$

17) $6n^2 + 5n - 6$

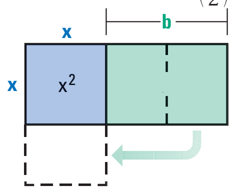
18) $16b^2 + 60b - 100$

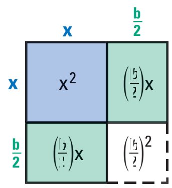
Lesson 10: Completing the Square

Completing the Square

words To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.

diagrams In each diagram, the combined area of the shaded regions $x^2 + bx$. Adding $\left(\frac{b}{2}\right)^2$ completes the square in the second diagram.





algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)\left(x + \frac{b}{2}\right) = \left(x + \frac{b}{2}\right)^2$

Find the value of c that makes $x^2 + 16x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

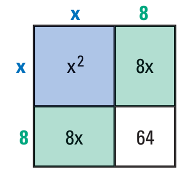
Solution

STEP 1 **Find** half the coefficient of x. $\frac{16}{2} = 8$

STEP 2 **Square** the result of Step 1. $8^2 = 64$

STEP 3 **Replace** c with the result of Step 2. $x^2 + 16x + 64$

▶ The trinomial $x^2 + 16x + c$ is a perfect square when $c = 64$. Then $x^2 + 16x + 64 = (x + 8)(x + 8) = (x + 8)^2$.



Vocabulary

To **complete the square** for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.

When you complete the square, $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

EXAMPLE 1 Make a perfect square trinomial

Find the value of c that makes $x^2 - 10x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

Solution

STEP 1 **Find** half the coefficient of x. $\frac{-10}{2} = -5$

STEP 2 **Square** the result of Step 1. $(-5)^2 = 25$

STEP 3 **Replace** c with the result of Step 2. $x^2 - 10x + 25$

The trinomial $x^2 - 10x + c$ is a perfect square when $c = 25$.
So, $x^2 - 10x + 25 = (x - 5)(x - 5) = (x - 5)^2$.

Exercises for Example 1

Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

1. $x^2 + 12x + c$ **2.** $x^2 - 18x + c$ **3.** $x^2 - 40x + c$

EXAMPLE 2 Solve $ax^2 + bx + c = 0$ when $a = 1$

Solve $x^2 - 16x + 8 = 0$ by completing the square.

Solution

$x^2 - 16x + 8 = 0$	Write original equation.
$x^2 - 16x = -8$	Write left side in the form $x^2 + bx$.
$x^2 - 16x + 64 = -8 + 64$	Add $\left(\frac{-16}{2}\right)^2 = 64$ to each side.
$(x - 8)^2 = 56$	Write left side as a binomial squared.
$x - 8 = \pm\sqrt{56}$	Take square roots of each side.
$x = 8 \pm \sqrt{56}$	Solve for x.
$x = 8 \pm 2\sqrt{14}$	Simplify: $\sqrt{56} = \sqrt{4} \cdot \sqrt{14} = 2\sqrt{14}$

The solutions are $8 + 2\sqrt{14}$ and $8 - 2\sqrt{14}$.

4.7 Study Guide *continued*

For use with pages 283–291

EXAMPLE 3 Solve $ax^2 + bx + c = 0$ when $a \neq 1$

Solve $3x^2 + 6x + 15 = 0$ by completing the square.

$3x^2 + 6x + 15 = 0$	Write original equation.
$x^2 + 2x + 5 = 0$	Divide each side by the coefficient of x^2, 3.
$x^2 + 2x = -5$	Write left side in the form $x^2 + bx$.
$x^2 + 2x + 1 = -5 + 1$	Add $\left(\frac{2}{2}\right)^2 = 1^2 = 1$ to each side.
$(x + 1)^2 = -4$	Write left side as a binomial squared.
$x + 1 = \pm\sqrt{-4}$	Take square roots of each side.
$x + 1 = \pm 2i$	Write in terms of the imaginary unit i.
$x = -1 \pm 2i$	Solve for x.

The solutions are $-1 + 2i$ and $-1 - 2i$.

Exercises for Examples 2 and 3

Solve the equation by completing the square.

4. $x^2 - 10x + 6 = 0$ **5.** $2x^2 + 16x + 8 = 0$ **6.** $5x^2 - 10x + 30 = 0$

EXAMPLE 4 Find the maximum value of a quadratic function

Revenue A retailer's revenue is modeled by $R = (300 + 10x)(50 - x)$. Rewrite in vertex form to find the number of units x that maximizes the revenue R.

Solution

$R = (300 + 10x)(50 - x)$	Write original function.
$R = 15{,}000 - 300x + 500x - 10x^2$	Use FOIL.
$R = -10x^2 + 200x + 15{,}000$	Combine like terms.
$R = -10(x^2 - 20x) + 15{,}000$	Prepare to complete the square.
$R = -10\left[x^2 - 20x + \left(\frac{-20}{2}\right)^2\right] + 10\left(\frac{-20}{2}\right)^2 + 15{,}000$	Add and subtract $10\left(\frac{-20}{2}\right)$.
$R = -10(x - 10)^2 + 16{,}000$	Write a perfect square trinomial as the square of a binomial.

The vertex is (10, 16,000), so the number of units that maximizes R is 10.

Exercises for Example 4

Write the equation in vertex form and identify the vertex.

7. $y = x^2 - 12x + 38$ **8.** $y = x^2 - 14x + 50$ **9.** $y = 2x^2 + 12x + 13$

10. Rework Example 4 where $R = (200 + 10x)(40 - x)$.

LESSON 4.8

Study Guide

For use with pages 292–299

GOAL **Solve quadratic equations using the quadratic formula.**

Vocabulary

The **quadratic formula:** Let a, b, and c be real numbers where $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

In the quadratic formula, the expression $b^2 - 4ac$ is called the **discriminant** of the associated equation $ax^2 + bx + c = 0$.

EXAMPLE 1 Solve a quadratic equation with two real solutions

Solve $x^2 - 5x = 4$.

$x^2 - 5x = 4$	Write original equation.
$x^2 - 5x - 4 = 0$	Write in standard form.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula
$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-4)}}{2(1)}$	$a = 1, b = -5, c = -4$
$x = \frac{5 \pm \sqrt{41}}{2}$	Simplify.

The solutions are $x = \frac{5 + \sqrt{41}}{2} \approx 5.70$ and $x = \frac{5 - \sqrt{41}}{2} \approx -0.70$.

EXAMPLE 2 Solve a quadratic equation with one real solution

Solve $4x^2 + 10x = -10x - 25$.

$4x^2 + 10x = -10x - 25$	Write original equation.
$x^2 + 20x + 25 = 0$	Write in standard form.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula
$x = \frac{-20 \pm \sqrt{20^2 - 4(4)25}}{2(4)}$	$a = 4, b = 20, c = 25$
$x = \frac{-20 \pm 0}{8}$	Simplify.
$x = -\frac{5}{2}$	Simplify.

The solution is $-\frac{5}{2}$.

4.8 Study Guide *continued*

For use with pages 292–299

EXAMPLE 3 Solve a quadratic equation with imaginary solutions

Solve $x^2 - 6x = -10$.

$x^2 - 6x = -10$	Write original equation.
$x^2 - 6x + 10 = 0$	Write in standard form.
$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)10}}{2(1)}$	$a = 1, b = -6, c = 10$
$x = \frac{6 \pm \sqrt{-4}}{2}$	Simplify.
$x = \frac{6 \pm 2i}{2}$	Rewrite using the imaginary unit i.
$x = 3 \pm i$	Simplify.

The solutions are $3 + i$ and $3 - i$.

Exercises for Examples 1, 2, and 3

Use the quadratic formula to solve the equation.

1. $x^2 + 4x = 2$
2. $2x^2 - 8x = 1$
3. $4x^2 + 2x = -2x - 1$
4. $16x^2 - 20x = 4x - 9$
5. $x^2 - 4x + 5 = 0$
6. $x^2 - x = -7$

EXAMPLE 4 Use the discriminant

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

a. $x^2 + 6x + 11$ **b.** $x^2 + 6x + 9$ **c.** $x^2 + 6x + 5$

Solution

	Equation	Discriminant	Solution(s)
	$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
a.	$x^2 + 6x + 11 = 0$	$6^2 - 4(1)(11) = -8$	Two imaginary: $-3 \pm i\sqrt{2}$
b.	$x^2 + 6x + 9 = 0$	$6^2 - 4(1)(9) = 0$	One real: -3
c.	$x^2 + 6x + 5 = 0$	$6^2 - 4(1)(5) = 16$	Two real: $-5, -1$

Exercises for Example 4

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

7. $x^2 - 2x - 1$
8. $x^2 - 12x + 36$
9. $x^2 + 7x + 14$

Exponents
(and their properties)

Pertinent Properties of Exponents

Power times a Power - when two powers *WITH THE SAME BASE* are multiplied together, we add their exponents

$$x^2 \bullet x^3 = x^{2+3} = x^5$$

power, times, power

Quotient of two Powers - when a power is divided by a second power *WITH THE SAME BASE*, we subtract their exponents

$$\frac{x^3}{x^2} = x^{3-2} = x^1$$

power, quotient, power

Power to a Power - When one power is being raised to a second power, we multiply the exponents

$$\left(x^3\right)^2 = x^{3\bullet 2} = x^6$$

power, power

Zero Power - Any base to the power of zero is 1

$$x^0 = 1$$

zero, power

Negative Exponents - A negative exponent means we need to take the reciprocal of that base (making the exponent positive)

$$\frac{3}{x^{-2}} = \frac{3x^2}{1} \quad \text{or} \quad x^{-2} = \frac{1}{x^2}$$

negative exponent

In Calculus, negative exponents are OK and often encouraged

And a bonus...

Distributive Property: A product to a power is equal to a product of the powers

$$(a \bullet b)^2 = a^2 \bullet b^2$$

And not really a rule but a useful tool for rewriting radicals...

Rewriting fractional exponents: If there is a fraction as an exponent, the denominator represents taking that root of the base (or of the power)

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Lesson 12: Exponent Properties

The deeper understanding of exponent properties goes back to what it means to be an exponent.

The exponent is merely a way to represent repeated multiplication by the same number (or variable expression)

so 4^3 means 4*4*4 (4 is multiplied 3 times)

Power times a Power:

$$x^2 \bullet x^3 = (x \bullet x) \bullet (x \bullet x \bullet x)$$ -definition of exponent

$$= x \bullet x \bullet x \bullet x \bullet x$$ -all the operations are the same so we don't need parentheses

$$= x^5$$ -definition of exponent

Quotient of two Powers:

$$\frac{x^3}{x^2} = \frac{x \bullet x \bullet x}{x \bullet x}$$ -definition of exponent

$$= \left(\frac{x}{x}\right) \bullet \left(\frac{x}{x}\right) \bullet x$$ -all the operations are the same (division is multiplication by the reciprocal), so we can associate however we would like

$$= 1 \bullet 1 \bullet x$$ -division to 1

$$= x^1$$ -definition of exponent

Power to a Power:

$$\left(x^3\right)^2 = \left(x^3\right) \bullet \left(x^3\right)$$ -definition of exponent

$$= (x \bullet x \bullet x) \bullet (x \bullet x \bullet x)$$ -definition of exponent

$$= x \bullet x \bullet x \bullet x \bullet x \bullet x$$ -all the operations are the same so we don't need parentheses

$$= x^6$$ -definition of exponent

Distributive Property:

$$(a \bullet b)^2 = (a \bullet b) \bullet (a \bullet b)$$ -definition of exponent

$$= a \bullet b \bullet a \bullet b$$ -all the operations are the same so we don't need parentheses

$$= a \bullet a \bullet b \bullet b$$ -all the operations are the same so we can move the factors

$$= a^2 \bullet b^2$$ -definition of exponent

All of these can be generalized to any exponent with any base.

These other two rules are derivations of the quotient of a power

Zero Power:

$$\frac{x^1}{x^1} = x^{1-1} = x^0$$ - Quotient of powers

$$\frac{x^1}{x^1} = \frac{1}{1} = 1$$ - Division to 1

$$x^0 = 1$$ - Substitution

Negative exponents:

$$\frac{x^1}{x^2} = x^{1-2} = x^{-1}$$ - Quotient of powers

$$\frac{x^1}{x^2} = \frac{x}{x \bullet x} = \frac{1}{x}$$ - Definition of exponent and Division to 1

$$x^{-1} = \frac{1}{x}$$ - Substitution

The connection between radicals and fractional exponents is a derivation of the power of a power

$$x^2 = a$$

$$\left(x^2\right)^{\frac{1}{2}} = a^{\frac{1}{2}}$$ - Raise both sides to the 1/2 power

$$x = a^{\frac{1}{2}}$$ - Power of a Power

$$x^2 = a$$

$$\sqrt{x^2} = \sqrt{a}$$

- Root of the power

$$x = \sqrt{a}$$

$$a^{\frac{1}{2}} = \sqrt{a}$$ - Substitution

While only shown with the 1/2 power, a similar argument can be made for any fractional exponent

LESSON **5.1** **Study Guide** *continued*
For use with pages 330–335

EXAMPLE 3 **Simplify an expression**

Simplify the expression. Tell which properties of exponents you used.

$\dfrac{y^7z^4}{(z^{-2})^{-1}z^2} = \dfrac{y^7z^4}{z^2z^2}$	Power of a power property
$= \dfrac{y^7z^4}{z^4}$	Product of powers property
$= y^7z^0$	Quotient of powers property
$= y^7 \cdot 1$	Zero exponent property
$= y^7$	Identity property of multiplication

EXAMPLE 4 **Compare real-life volumes**

Softball The radius of a softball is about $\frac{4}{3}$ times the radius of a baseball. How many times as great as the baseball's volume is the softball's volume?

Solution

Let r represent the radius of a baseball. Then $\frac{4}{3}r$ represents the radius of a softball.

$\dfrac{\text{softball's volume}}{\text{baseball's volume}} = \dfrac{\frac{4}{3}\pi\left(\frac{4}{3}r\right)^3}{\frac{4}{3}\pi r^3}$	The volume of a sphere is $\frac{4}{3}\pi r^3$.
$= \dfrac{\cancel{\frac{4}{3}\pi}\left(\frac{4}{3}\right)^3 r^3}{\cancel{\frac{4}{3}\pi} r^3}$	Power of a product property
$= \dfrac{64}{27}r^0$	Power of a quotient and quotient of powers
$= \dfrac{64}{27} \cdot 1$	Zero exponent property
≈ 2.370370	Use a calculator.

The volume of a softball is about 2.37 times as great as the volume of a baseball.

Exercises for Examples 3 and 4

Simplify the expression. Tell which properties of exponents you used.

6. $t^7t^2t^{-8}$ **7.** $(k^{-3}m^4)^{-2}$ **8.** $\left(\dfrac{f^5}{g^{-2}}\right)^{-3}$ **9.** $\left(\dfrac{3x}{z^2}\right)^0$

10. Rework Example 4 where the radius of a volleyball is about 3 times the radius of a baseball.

LESSON 6.2

Study Guide

For use with pages 420–427

GOAL **Simplify expressions involving rational exponents.**

Vocabulary

A radical with index n is in **simplest form** if the radicand has no perfect nth powers as factors and any denominator has been rationalized.

Two radical expressions with the same index and radicand are **like radicals.**

EXAMPLE 1 Use properties of exponents

Use the properties of rational exponents to simplify the expression.

a. $(8^2 3^2)^{-1/2} = [(8 \cdot 3)^2]^{-1/2} = (24^2)^{-1/2} = 24^{-1} = \frac{1}{24}$

b. $(4^{1/5} \cdot 2^{1/5})^{10} = (4^{1/5})^{10} \cdot (2^{1/5})^{10} = 4^2 \cdot 2^2 = 16 \cdot 4 = 64$

c. $\frac{5^{3/4}}{5^{1/2}} \cdot 5^2 = 5^{3/4 - 1/2} \cdot 5^2 = 5^{1/4} \cdot 5^2 = 5^{1/4 + 2} = 5^{9/4}$

EXAMPLE 2 Use properties of radicals

Use the properties of radicals to simplify the expression.

a. $\sqrt{8} \cdot \sqrt{50} = \sqrt{8 \cdot 50} = \sqrt{400} = 20$ Product property

b. $\frac{\sqrt[3]{192}}{\sqrt[3]{24}} = \frac{\sqrt[3]{64 \cdot 3}}{\sqrt[3]{8 \cdot 3}} = \frac{4\sqrt[3]{3}}{2\sqrt[3]{3}} = 2$ Quotient property

EXAMPLE 3 Write radicals in simplest form

a. $\sqrt[5]{64} = \sqrt[5]{32 \cdot 2}$ Factor out perfect fifth powers.

$= \sqrt[5]{32} \cdot \sqrt[5]{2}$ Product property

$= 2\sqrt[5]{2}$ Simplify.

b. $\frac{\sqrt[3]{3}}{\sqrt[3]{4}} = \frac{\sqrt[3]{3}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$ Make denominator a perfect cube.

$= \frac{\sqrt[3]{6}}{\sqrt[3]{8}}$ Product property

$= \frac{\sqrt[3]{6}}{2}$ Simplify.

LESSON **6.2** **Study Guide** *continued*
For use with pages 420–427

EXAMPLE 4

Add and subtract like radicals and roots

Simplify the expression.

a. $\sqrt[4]{162} - \sqrt[4]{32} = \sqrt[4]{81} \cdot \sqrt[4]{2} - \sqrt[4]{16} \cdot \sqrt[4]{2} = 3\sqrt[4]{2} - 2\sqrt[4]{2} = (3 - 2)\sqrt[4]{2} = \sqrt[4]{2}$

b. $7^{3/2} + 4(7^{3/2}) = (1 + 4)7^{3/2} = 5(7^{3/2})$

Exercises for Examples 1, 2, 3, and 4

Simplify the expression.

1. $(8^{1/2} \cdot 9^{1/4})^2$ **2.** $\left(\frac{10}{10^{2/3}}\right)^2$ **3.** $125^{-1/3}$ **4.** $11^{-1/2} \cdot 11^{5/2}$

5. $\sqrt{18} \cdot \sqrt{27}$ **6.** $\sqrt[3]{16} \cdot \sqrt[3]{24}$ **7.** $\frac{\sqrt{200}}{\sqrt{8}}$ **8.** $\frac{\sqrt[5]{160}}{\sqrt[5]{5}}$

9. $\sqrt[3]{432}$ **10.** $\frac{\sqrt[4]{2}}{\sqrt[4]{3}}$ **11.** $3(6)^{1/4} + 5(6)^{1/4}$ **12.** $9(54)^{1/2} - 54^{1/2}$

EXAMPLE 5

Simplify expressions involving variables

Simplify the expression. Assume all variables are positive.

a. $\sqrt[3]{\frac{8y^6}{z^{12}}} = \frac{\sqrt[3]{8y^6}}{\sqrt[3]{z^{12}}} = \frac{\sqrt[3]{(2y^2)^3}}{\sqrt[3]{(z^4)^3}} = \frac{2y^2}{z^4}$

b. $\sqrt[3]{\frac{a^2}{b}} = \sqrt[3]{\frac{a^2 \cdot b^2}{b \cdot b^2}} = \sqrt[3]{\frac{a^2b^2}{b^3}} = \frac{\sqrt[3]{a^2b^2}}{\sqrt[3]{b^3}} = \frac{\sqrt[3]{a^2b^2}}{b}$

EXAMPLE 6

Add and subtract expressions involving variables

Perform the indicated operation. Assume all variables are positive.

a. $-4\sqrt{k} - 5\sqrt{k} = (-4 - 5)\sqrt{k} = -9\sqrt{k}$

b. $9a^{1/3}b - 2a^{1/3}b = (9 - 2)a^{1/3}b = 7a^{1/3}b$

Exercises for Examples 5 and 6

Simplify the expression. Assume all variables are positive.

13. $\sqrt[4]{\frac{81m^4}{16n^8}}$ **14.** $(8p^6q^3)^{2/3}$ **15.** $\sqrt[3]{\frac{2x}{(3y)^2}}$

16. $6\sqrt{h} - 8\sqrt{h}$ **17.** $11a^{1/2}b - 4a^{1/2}b$ **18.** $3\sqrt{5y^4} - y\sqrt{20y^2}$

LESSON 5.3

Study Guide

For use with pages 346–352

GOAL **Add, subtract, and multiply polynomials.**

EXAMPLE 1 Add and subtract polynomials

a. Add $x^3 - 2x + 8$ and $7x^2 + 2x - 5$ in a vertical format.

$$\begin{array}{rrrr} x^3 & & -\,2x & +\,8 \\ + & 7x^2 & +\,2x & -\,5 \\ \hline x^3 & +\,7x^2 & & +\,3 \end{array}$$

b. Add $7x^2 + 2x - 5$ and $x^3 - 2x^2 + 4x + 8$ in a horizontal format.

$$(7x^2 + 2x - 5) + (x^3 - 2x^2 + 4x + 8) = x^3 + 7x^2 - 2x^2 + 2x + 4x - 5 + 8$$
$$= x^3 + 5x^2 + 6x + 3$$

c. Subtract $y^2 - 2y + 7$ from $2y^2 + 6y - 2$ in a vertical format. Align like terms, then add the opposite of the subtracted polynomial.

$$\begin{array}{r} 2y^2 + 6y - 2 \\ -(\;y^2 - 2y + 7) \end{array} \quad \longrightarrow \quad \begin{array}{r} 2y^2 + 6y - 2 \\ + \; -y^2 + 2y - 7 \\ \hline y^2 + 8y - 9 \end{array}$$

d. Subtract $6x^3 + 5x^2 - 7x$ from $5x^3 - x^2 + 10x$ in a horizontal format. Write the opposite of the subtracted polynomial, then add like terms.

$$(5x^3 - x^2 + 10x) - (6x^3 + 5x^2 - 7x) = 5x^3 - x^2 + 10x - 6x^3 - 5x^2 + 7x$$
$$= -x^3 - 6x^2 + 17x$$

Exercises for Example 1

Find the sum or difference.

1. $(4x^3 - 2x^2 + 5) + (-x^3 - x^2 + 4x - 2)$
2. $(9x^2 - 8x + 3) - (2x^2 + x - 4)$

EXAMPLE 2 Multiply polynomials vertically and horizontally

a. Multiply $-x^2 - 2x + 7$ and $x + 2$ in a vertical format.

$$\begin{array}{rl} -x^2 - 2x + 7 & \\ \times \qquad\qquad x + 2 & \\ \hline -2x^2 - 4x + 14 & \text{Multiply } -x^2 - 2x + 7 \text{ by } 2. \\ -x^3 - 2x^2 + 7x \qquad & \text{Multiply } -x^2 - 2x + 7 \text{ by } x. \\ \hline -x^3 - 4x^2 + 3x + 14 & \text{Combine like terms.} \end{array}$$

b. Multiply $(x + 3)(x - 1)(x + 2)$ in a horizontal format.

$$(x + 3)(x - 1)(x + 2) = (x^2 + 2x - 3)(x + 2)$$
$$= (x^2 + 2x - 3)x + (x^2 + 2x - 3)2$$
$$= x^3 + 2x^2 - 3x + 2x^2 + 4x - 6$$
$$= x^3 + 4x^2 + x - 6$$

LESSON **5.3**

Study Guide *continued*

For use with pages 346–352

Exercises for Example 2

Find the product.

3. $(z^2 - 5z + 3)(z - 1)$

4. $(x - 2)(x - 1)(x + 3)$

EXAMPLE 3

Use special product patterns

a. $(6a + 1)^2 = (6a)^2 + 2(6a)(1) + 12$ — Square of a binomial

$= 36a^2 + 12a + 1$

b. $(5z + 2)(5z - 2) = (5z)^2 - 22$ — Sum and difference

$= 25z^2 - 4$

c. $(2y + 3)^3 = (2y)^3 + 3(2y)^2(3) + 3(2y)(3)^2 + 3^3$ — Cube of a binomial

$= 8y^3 + 36y^2 + 54y + 27$

Exercises for Example 3

Find the product.

5. $(x + 2)^3$

6. $(7y - 2)^2$

7. $(4d + 3)(4d - 3)$

8. $(2a + 5)^2$

EXAMPLE 4

Use polynomial models

Volume of a Cylinder The volume of a cylinder is modeled by $V = \pi r^2 h$ where r is the radius of the circular base and h is the height of the cylinder. Write the volume of the cylinder as a polynomial in standard form when $r = (x + 1)$ and $h = (x + 2)$. Then find the volume when $x = 1$.

Solution

$V = \pi r^2 h$	Formula for volume of a cylinder
$= \pi(x + 1)^2(x + 2)$	Substitute $r = x + 1$ and $h = x + 2$.
$= \pi(x^2 + 2x + 1)(x + 2)$	Square of a binomial
$= \pi(x^3 + 2x^2 + x + 2x^2 + 4x + 2)$	Distributive property
$= \pi(x^3 + 4x^2 + 5x + 2)$	Combine like terms.

The volume is written as the product of π and a polynomial written in standard form. When $x = 1$, $V = \pi\left[1^3 + 4(1)^2 + 5(1) + 2\right] = 12\pi$.

Exercise for Example 4

9. Rework Example 4 where $r = (x + 2)$ and $h = (x + 3)$. Then find the volume when $x = 1$.

LESSON 7.1

Study Guide

For use with pages 432–439

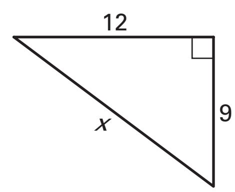
GOAL **Find side lengths in right triangles.**

> **Vocabulary**
>
> **Theorem 7.1 Pythagorean Theorem:** In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.
>
> A **Pythagorean triple** is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

EXAMPLE 1

Find the length of a hypotenuse

Find the length of the hypotenuse of the right triangle.



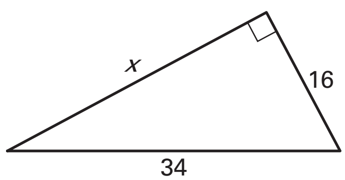
Solution

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Pythagorean Theorem
$x^2 = 9^2 + 12^2$	Substitute.
$x^2 = 81 + 144$	Multiply.
$x^2 = 225$	Add.
$x = 15$	Find the positive square root.

Exercises for Example 1

Identify the unknown side as a *leg* or *hypotenuse*. Then, find the unknown side length of the right triangle. Write your answer in simplest radical form.

1.



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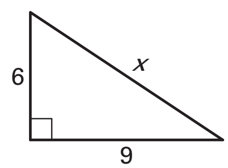
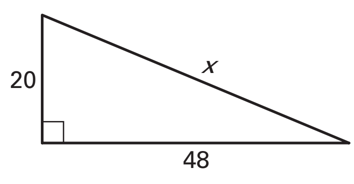


3.



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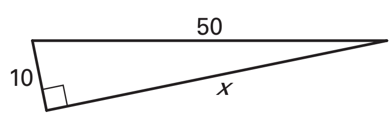

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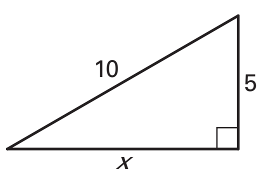

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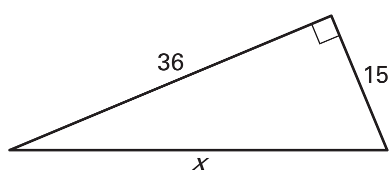


LESSON 7.2

Study Guide

For use with pages 440–447

GOAL **Use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle.**

Vocabulary

Theorem 7.2 Converse of the Pythagorean Theorem: If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

Theorem 7.3: If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an acute triangle.

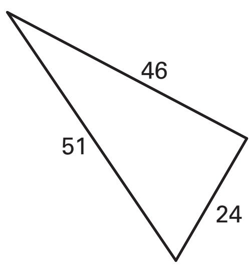
Theorem 7.4: If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is an obtuse triangle.

EXAMPLE 1

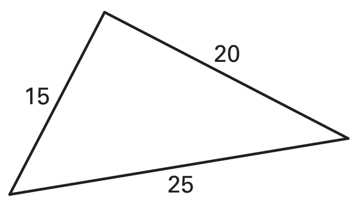
Verify right triangles

Tell whether the given triangle is a right triangle.

a.



b.



Solution

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $51^2 \stackrel{?}{=} 24^2 + 46^2$

$2601 \stackrel{?}{=} 576 + 2116$

$2601 \neq 2692$

The triangle is not a right triangle.

b. $25^2 \stackrel{?}{=} 15^2 + 20^2$

$625 \stackrel{?}{=} 225 + 400$

$625 = 625$ ✓

The triangle is a right triangle.

LESSON **7.2** **Study Guide** *continued*
For use with pages 440–447

EXAMPLE 2 **Classify triangles**

Can segments with lengths of 22 inches, 43 inches, and 49 inches form a triangle? If so, would the triangle be *acute*, *right*, or *obtuse*?

Solution

STEP 1 **Use** the Triangle Inequality Theorem to check that the segments can make a triangle.

$22 + 43 \overset{?}{>} 49$ $\quad$ $22 + 49 \overset{?}{>} 43$ $\quad$ $43 + 49 \overset{?}{>} 22$

$65 > 49$ ✓ $\quad$ $71 > 43$ ✓ $\quad$ $92 > 22$ ✓

The side lengths 22 inches, 43 inches, and 49 inches can form a triangle.

STEP 2 **Classify** the triangle by comparing the square of the length of the longest side with the sum of the squares of the lengths of the shorter sides.

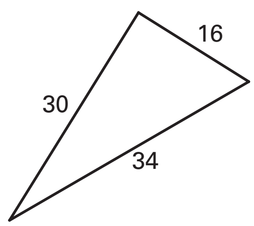
$c^2 \ \underline{\ ?\ }\ a^2 + b^2$	Compare c^2 with $a^2 + b^2$.
$49^2 \ \underline{\ ?\ }\ 22^2 + 43^2$	Substitute.
$2401 \ \underline{\ ?\ }\ 484 + 1849$	Simplify.
$2401 > 2333$	c^2 is greater than $a^2 + b^2$.

The side lengths 22 inches, 43 inches, and 49 inches form an obtuse triangle.

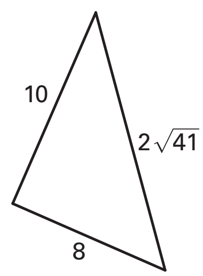
Exercises for Examples 1 and 2

Tell whether a triangle with the given side lengths is a right triangle.

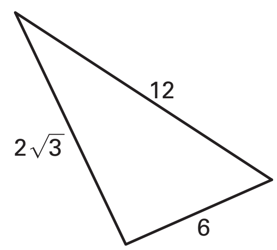
1.



2.



3.



Tell whether segments with the given side lengths can form a triangle. If so, classify the triangle as *acute*, *right*, or *obtuse*.

4. 10.5, 36, 37.5 $\quad$ **5.** 8, 21, 27 $\quad$ **6.** 18, 23, 25.2

7. 30, 40, 60 $\quad$ **8.** 12.3, 15.2, 23.5 $\quad$ **9.** 20, 37.5, 42.5

LESSON 7.7

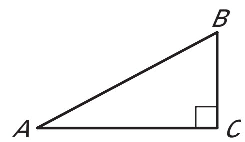
Study Guide

For use with pages 483–489

GOAL **Use inverse tangent, sine, and cosine ratios.**

Vocabulary

To **solve a right triangle** means to find the measures of all of its sides and angles.



Inverse Trigonometric Ratios:
Let $\angle A$ be an acute angle.

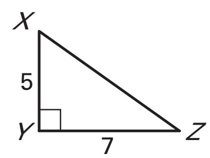
Inverse Tangent: If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine: If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine: If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\cos^{-1} \frac{AC}{AB} = m\angle A$

EXAMPLE 1 Use an inverse tangent to find an angle measure

Use a calculator to approximate the measure of $\angle X$ to the nearest tenth of a degree.



Because $\tan X = \frac{7}{5} = 1.4$, $\tan^{-1} 1.4 = m\angle X$. Use a calculator.

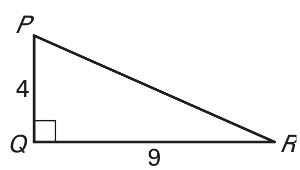
$\tan^{-1} 1.4 \approx 54.46232221\ldots$

So, the measure of $\angle X$ is approximately 54.5°.

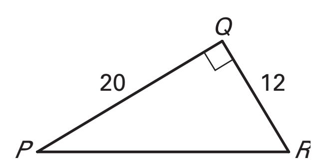
Exercises for Example 1

Use a calculator to approximate the measure of $\angle P$ to the nearest tenth of a degree.

1.



2.



EXAMPLE 2 Use an inverse sine and an inverse cosine

Let $\angle A$ and $\angle B$ be acute angles in a right triangle. Use a calculator to approximate the measure of $\angle A$ and $\angle B$ to the nearest tenth of a degree.

a. $\sin A = 0.19$

b. $\cos B = 0.56$

Solution

a. $m\angle A = \sin^{-1} 0.19 \approx 11.0°$

b. $m\angle B = \cos^{-1} 0.56 \approx 55.9°$

LESSON 7.7

Study Guide *continued*

For use with pages 483–489

Exercises for Example 2

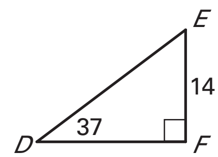
Let $\angle A$ be an acute angle in a right triangle. Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

3. $\sin A = 0.95$ **4.** $\sin A = 0.23$ **5.** $\cos A = 0.12$ **6.** $\cos A = 0.67$

EXAMPLE 3

Solve a right triangle

Solve the right triangle. Round decimal answers to the nearest tenth.



STEP 1 **Find** $m\angle E$ by using the Triangle Sum Theorem.

$$180° = 90° + 37° + m\angle E$$

$$53° = m\angle E$$

STEP 2 **Approximate** DF by using a tangent ratio.

$\tan 37° = \frac{14}{DF}$ Write ratio for tangent of 37°.

$DF = \frac{14}{\tan 37°}$ Solve for DF.

$DF \approx \frac{14}{0.7536} \approx 18.6$ Approximate tan 37°, simplify, and round.

STEP 3 **Approximate** DE by using a sine ratio.

$\sin 37° = \frac{14}{DE}$ Write ratio for sine of 37°.

$DE = \frac{14}{\sin 37°}$ Solve for DE.

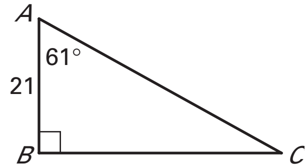
$DE \approx \frac{14}{0.6018} \approx 23.3$ Approximate sin 37°, simplify, and round.

The angle measures are 37°, 53°, and 90°. The side lengths are 14, about 18.6, and about 23.3.

Exercises for Example 3

Solve the right triangle. Round decimal answers to the nearest tenth.

7.



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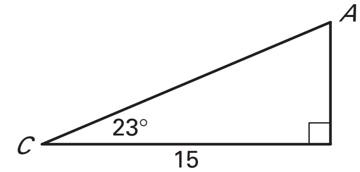


LESSON 13.1

Study Guide

For use with pages 852–858

GOAL **Use trigonometric functions to find lengths.**

Vocabulary

Right Triangle Definitions of Trigonometric Functions

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as follows:

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$ $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$ $\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

The abbreviations *opp*, *adj*, and *hyp* are often used to represent the side lengths of the right triangle. Note that the ratios in the second row are reciprocals of the ratios in the first row:

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Trigonometric Values for Special Angles

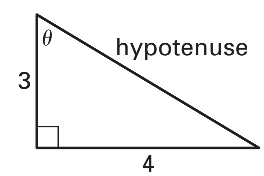
The table below gives the values of the six trigonometric functions for the angles 30°, 45°, and 60°.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

EXAMPLE 1

Evaluate trigonometric functions

Evaluate the six trigonometric functions of the angle *θ*.



Solution

From the Pythagorean theorem, the length of the hypotenuse is $\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$ $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$ $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$ $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$

LESSON 13.1

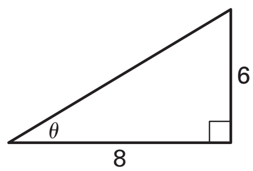
Study Guide *continued*

For use with pages 852–858

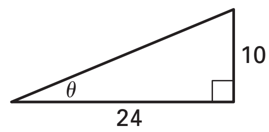
Exercises for Example 1

Evaluate the six trigonometric functions of the angle θ.

1.



2.



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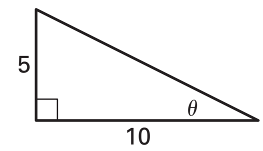
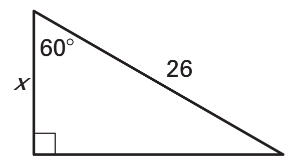


EXAMPLE 2 Find an unknown side length of a right triangle

Find the value of *x* for the right triangle shown.



Solution

Because you are given the side adjacent to $\theta = 60°$ and the hypotenuse, you can write an equation using a trigonometric function that involves the ratio of x and 26. Then solve the equation for x.

$\cos 60° = \frac{\text{adj}}{\text{hyp}}$ Write trigonometric equation.

$\frac{1}{2} = \frac{x}{26}$ Substitute $\frac{1}{2}$ for cos 60°, x for adj, and 26 for hyp.

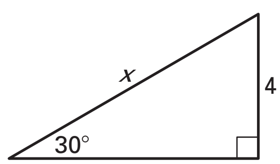
$13 = x$ Multiply each side by 26.

The length of the side is $x = 13$.

Exercises for Example 2

Find the value of *x* for the right triangle shown.

4.



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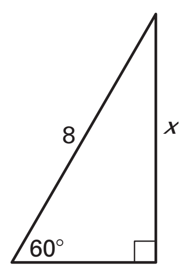

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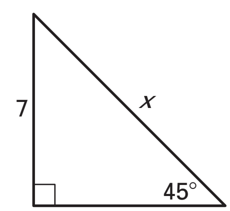


LESSON 13.5

Study Guide

For use with pages 881–888

GOAL **Solve triangles that have no right angle.**

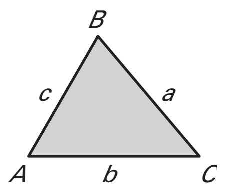
Vocabulary

The **law of sines** can be written in either of the following forms for $\triangle ABC$ with sides of length a, b, and c.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a Triangle

The area of any triangle is given by one half the product of the lengths of two sides times the sine of their included angle. For $\triangle ABC$ shown, there are three ways to calculate the area:

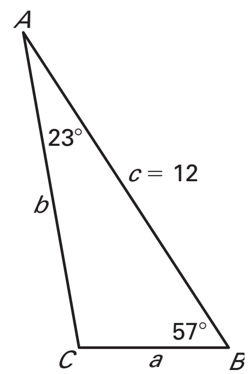


$$\text{Area} = \frac{1}{2}bc \sin A \qquad \text{Area} = \frac{1}{2}ac \sin B \qquad \text{Area} = \frac{1}{2}ab \sin C$$

EXAMPLE 1

Solve a triangle for the AAS or ASA case

Solve $\triangle ABC$ with $A = 23°$, $B = 57°$, and $c = 12$.



Solution

The sum of the angles of any triangle is 180°. So, you can find the measure of angle C as follows:

$C = 180° - 23° - 57° = 100°$

By the law of sines, you can write

$$\frac{a}{\sin 23°} = \frac{b}{\sin 57°} = \frac{12}{\sin 100°}$$

$\frac{a}{\sin 23°} = \frac{12}{\sin 100°}$	Write two equations, each with one variable.	$\frac{b}{\sin 57°} = \frac{12}{\sin 100°}$
$a = \frac{12 \sin 23°}{\sin 100°}$	Solve for each variable.	$b = \frac{12 \sin 57°}{\sin 100°}$
$a \approx 4.76$	Use a calculator.	$b \approx 10.22$

In $\triangle ABC$, $C = 100°$, $a \approx 4.76$, and $b \approx 10.22$.

Exercises for Example 1

Solve $\triangle ABC$.

1. $A = 23°, B = 57°, a = 12$

2. $B = 34°, C = 108°, b = 20$

3. $A = 41°, C = 77°, a = 10.5$

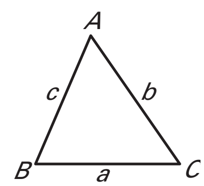
4. $B = 20°, C = 31°, b = 210$

LESSON 13.6
Study Guide
For use with pages 889–894

GOAL **Solve triangles using the law of cosines.**

Vocabulary

If $\triangle ABC$ has sides of length a, b, and c as



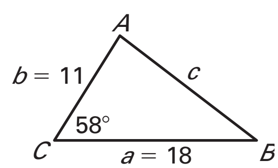
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Heron's Area Formula

The area of the triangle with sides of length a, b, and c is Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$. The variable s is called the *semiperimeter*, or half-perimeter, of the triangle.

EXAMPLE 1
Solve a triangle for the SAS case

Solve $\triangle ABC$ with $a = 18$, $b = 11$, and $C = 58°$.



Because you are not given a side opposite an angle, you cannot use the law of sines. Use the law of cosines to find side length c.

$c^2 = a^2 + b^2 - 2ab \cos C$	Law of cosines
$c^2 = 18^2 + 11^2 - 2(18)(11) \cos 58°$	Substitute for a, b, and C.
$c^2 \approx 235.2$	Simplify.
$c \approx \sqrt{235.2} \approx 15.3$	Take positive square root.

Because you know all three sides and an angle, you can use either the law of cosines or the law of sines to find the measure of a second angle. Use the law of sines to find the measure of angle A.

$\frac{\sin A}{a} = \frac{\sin C}{c}$	Law of sines
$\frac{\sin A}{18} = \frac{\sin 58°}{15.3}$	Substitute for a, c, and C.
$\sin A = \frac{18 \sin 58°}{15.3} \approx 0.9977$	Multiply each side by 18 and simplify.
$A \approx 86.1°$	Use inverse sine.

The third angle B of the triangle is $B \approx 180° - 86.1° - 58° = 35.9°$.

In $\triangle ABC$, $c \approx 15.3$, $A \approx 86.1°$, and $B \approx 35.9°$.

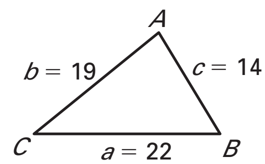
Exercise for Example 1

1. Solve $\triangle ABC$: $A = 62°$, $b = 56$, $c = 40$.

LESSON 13.6 **Study Guide** *continued*
For use with pages 889–894

EXAMPLE 2
Solve a triangle for the SSS case

Solve $\triangle ABC$ with $a = 22$, $b = 19$, and $c = 14$.



Solution

First find the angle opposite the longest side, $\overline{BC}$. Use the law of cosines to solve for A.

$$a^2 = b^2 + c^2 - 2bc \cos A \qquad \text{Law of cosines}$$

$$22^2 = 19^2 + 14^2 - 2(19)(14) \cos A \qquad \text{Substitute.}$$

$$\frac{22^2 - 19^2 - 14^2}{-2(19)(14)} = \cos A \qquad \text{Solve for } \cos A.$$

$$0.1372 \approx \cos A \qquad \text{Simplify.}$$

$$A \approx \cos^{-1} 0.1372 \approx 82.1° \qquad \text{Use inverse cosine.}$$

Now use the law of sines to find B.

$$\frac{\sin B}{b} = \frac{\sin A}{a} \qquad \text{Law of sines}$$

$$\frac{\sin B}{19} = \frac{\sin 82.1°}{22} \qquad \text{Substitute for } b, a, \text{ and } A.$$

$$\sin B = \frac{19 \sin 82.1°}{22} \qquad \text{Multiply each side by 19.}$$

$$\sin B \approx 0.8554 \qquad \text{Use a calculator.}$$

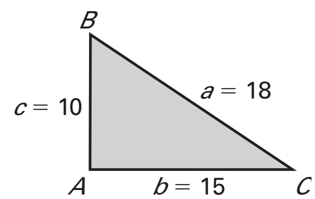
$$B \approx 58.8° \qquad \text{Use inverse sine.}$$

The third angle C of the triangle is $C \approx 180° - 82.1° - 58.8° = 39.1°$.

In $\triangle ABC$, $A \approx 82.1°$, $B \approx 58.8°$, and $C \approx 39.1°$.

EXAMPLE 3
Solve a multi-step problem

Find the area of $\triangle ABC$.



STEP 1 **Find** the semiperimeter s.

$$s = \frac{1}{2}(a + b + c) = \frac{1}{2}(18 + 15 + 10) = 21.5$$

STEP 2 **Use** Heron's Formula to find the area of $\triangle ABC$.

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$$
$$= \sqrt{21.5(21.5 - 18)(21.5 - 15)(21.5 - 10)} \approx 75$$

The area of $\triangle ABC$ is about 75 square units.

Exercise for Examples 2 and 3

2. Solve $\triangle ABC$. Then find the area of $\triangle ABC$.

$a = 39$, $b = 14$, $c = 27$

LESSON 11.1

Study Guide

For use with pages 720–726

GOAL **Find areas of triangles and parallelograms.**

Vocabulary

The **bases of a parallelogram** are either pair of the parallel sides.

The **height of a parallelogram** is the perpendicular distance between the bases.

Postulate 24 Area of a Square Postulate: The area of a square is the square of the length of its side.

Postulate 25 Area Congruence Postulate: If two polygons are congruent, then they have the same area.

Postulate 26 Area Addition Postulate: The area of a region is the sum of the areas of its nonoverlapping parts.

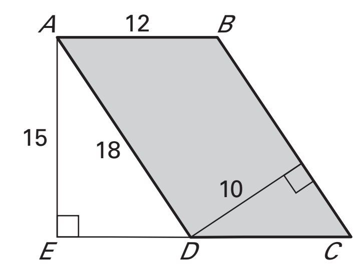
Theorem 11.1 Area of a Rectangle: The area of a rectangle is the product of its base and height.

Theorem 11.2 Area of a Parallelogram: The area of a parallelogram is the product of a base and its corresponding height.

Theorem 11.3 Area of a Triangle: The area of a triangle is one half the product of a base and its corresponding height.

EXAMPLE 1 Use a formula to find area

Find the area of ▱*ABCD*.



Solution

To find the area of the parallelogram, you can use either $\overline{AB}$ or $\overline{AD}$ as the base. If base $\overline{AB}$ is used, then the height is the perpendicular distance to the other base $\overline{DC}$. So, $b = 12$ and $h = 15$.

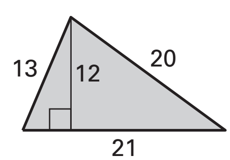
$A = bh = 12(15) = 180$

The area of ▱*ABCD* is 180 square units.

Exercises for Example 1

Find the area of the polygon.

1.



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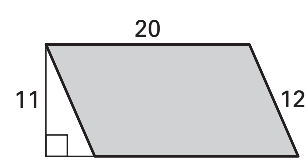
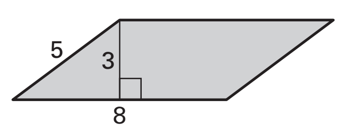


3.



LESSON **11.1** **Study Guide** *continued*
For use with pages 720–726

EXAMPLE 2 **Solve for unknown measure**

A triangle has an area of 126 square feet and a height of 14 feet. What is the length of the base?

Solution

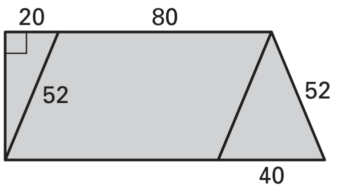
$126 = \frac{1}{2}b(14)$ Substitute 126 for A and 14 for h into the formula for the area of a triangle.

$18 = b$ Solve for b.

The length of the base of triangle is 18 feet.

EXAMPLE 3 **Solve a multi-step problem**

Grass Seed How many pounds of grass seed should you buy to cover the lawn shown at the right? A pound of Kentucky bluegrass seed covers 1650 square feet.



Solution

You can use a right triangle, a parallelogram, and a triangle to approximate the area of the lawn.

STEP 1 **Find** the height of the right triangle. The hypotenuse of the the right triangle has the same measure as the 52 foot side of the parallelogram.

$52^2 = 20^2 + h^2$ Pythagorean Theorem

$48 = h$ Solve for the positive value of h.

STEP 2 **Find** the approximate area of the lawn.

Area = Area of right triangle + Area of parallelogram + Area of triangle

$= \frac{1}{2}(20)(48) + (80)(48) + \frac{1}{2}(40)(48) = 5280 \text{ ft}^2$

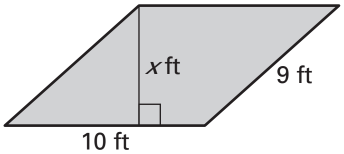
STEP 3 Determine how many pounds of seed you need.

$5280 \text{ ft}^2 \cdot \frac{1 \text{ lb}}{1650 \text{ ft}^2} = 3.2 \text{ lb}$ Use unit analysis.

You need to buy 4 pounds of grass seed so you will have enough.

Exercises for Examples 2 and 3

4. The parallelogram shown at the right has an area of 70 square feet. Find the value of x.



5. In Example 3, suppose a 10 foot by 35 foot rectangular driveway is constructed on a portion of the lawn. What is the approximate area you need to seed?

LESSON **11.6**

Study Guide

For use with pages 762–769

GOAL **Find areas of regular polygons inscribed in circles.**

Vocabulary

The **center of a polygon** is the center of its circumscribed circle.

The **radius of a polygon** is the radius of its circumscribed circle.

The **apothem of a polygon** is the distance from its center to any side of the polygon.

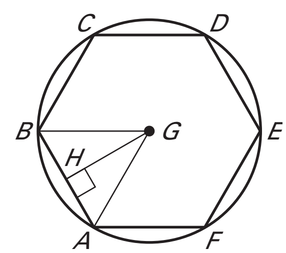
A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices of the polygon.

Theorem 11.11 Area of a Regular Polygon: The area of a regular *n*-gon with side length *s* is half the product of the apothem *a* and the perimeter *P*.

EXAMPLE 1

Find angle measures in a regular polygon

In the diagram, *ABCDEF* is a regular hexagon inscribed in ⊙*G*. Find each angle measure.



a. *m*≥ *AGB* **b.** *m*≥ *AGH* **c.** *m*≥ *HAG*

Solution

a. ≥ *AGB* is the central angle, so $m\geq AGB = \frac{360°}{6}$, or 60°.

b. $\overline{HG}$ is an apothem, which makes it an altitude of isosceles △*AGB*. So, $\overline{HG}$ bisects ≥ *AGB* and $m\geq AGH = \frac{1}{2}m\geq AGB = \frac{1}{2}(60°) = 30°$.

c. The sum of the measures of right △*HAG* is 180°. So, 90° + 30°+ *m*≥ *HAG* = 180°, and *m*≥ *HAG* = 60°.

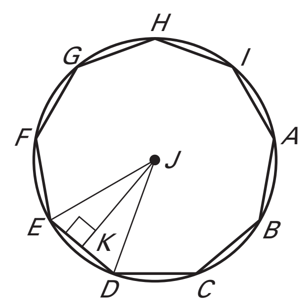
Exercises for Example 1

Find the given angle measure for the regular nonagon inscribed in ⊙*J*.

1. *m*≥ *DJE*

2. *m*≥ *DJK*

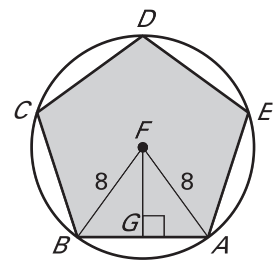
3. *m*≥ *JKD*



LESSON **11.6** **Study Guide** *continued*
For use with pages 762–769

EXAMPLE 2 Find the perimeter and area of a regular polygon

A regular pentagon is inscribed in a circle with radius 8 units. Find the perimeter and area of the pentagon.



Solution

The measure of the central ≥ AFB is $\frac{360°}{5}$, or 72°. Apothem $\overline{GF}$ bisects the central angle, so m≥ $AFG = 36°$. To find the lengths of the legs, use trigonometric ratios for right $\triangle AFG$.

Leg $\overline{AG}$:	$\sin 36° = \frac{AG}{AF}$	Use sine ratio.
	$\sin 36° = \frac{AG}{8}$	Substitute 8 for AF.
	$8 \cdot \sin 36° = AG$	Cross Product Property
Leg $\overline{FG}$:	$\cos 36° = \frac{FG}{AF}$	Use cosine ratio.
	$\cos 36° = \frac{FG}{8}$	Substitute 8 for AF.
	$8 \cdot \cos 36° = FG$	Cross Product Property

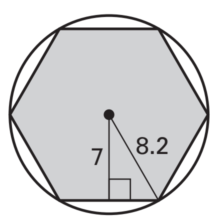
The regular hexagon has side length $s = 2AG = 2(8 \cdot \sin 36°) = 16 \cdot \sin 36°$ and apothem $a = FG = 8 \cdot \cos 36°$.

So, the perimeter is $P = 6s = 6(16 \cdot \sin 36°) = 96 \cdot \sin 36° \approx 56.43$ units, and the area is $A = \frac{1}{2}aP \approx \frac{1}{2}(8 \cdot \cos 36°)(96 \cdot \sin 36°) \approx 182.60$ square units.

Exercises for Example 2

Find the perimeter and the area of the regular polygon.

4.



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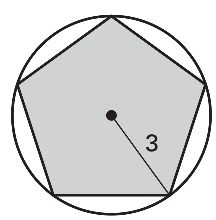

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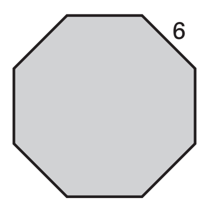


Lesson 19: Arc Length and Sector Area

11.4 *For use with pages 746–752*

GOAL **Find arc lengths and other measures of circles.**

Vocabulary

The **circumference** of a circle is the distance around the circle.

An **arc length** is a portion of the circumference of a circle.

Theorem 11.8 Circumference of a Circle: The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.

Arc Length Corollary: In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

EXAMPLE 1 Use the formula for circumference

Find the indicated measure.

a. Circumference of a circle with radius 11 feet

b. Diameter of a circle with circumference 75 meters

Solution

a.

$C = 2\pi r$	Write circumference formula.
$= 2 \cdot \pi \cdot 11$	Substitute 11 for r.
$= 22\pi$	Simplify.
≈ 69.08	Use a calculator.

The circumference is about 69.08 feet.

b.

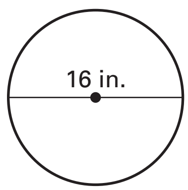
$C = \pi d$	Write circumference formula.
$75 = \pi d$	Substitute 75 for C.
$\frac{75}{\pi} = d$	Divide each side by π.
$23.87 \approx d$	Use a calculator.

The diameter is about 23.87 meters.

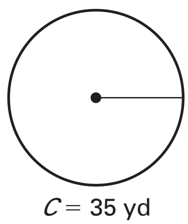
Exercises for Example 1

Use the diagram to find the indicated measure.

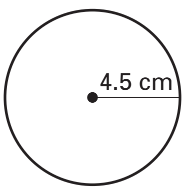
1. Circumference



2. Radius



3. Circumference

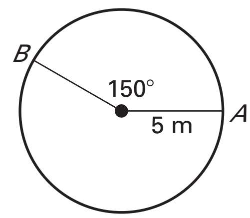


LESSON 11.4 **Study Guide** *continued*

For use with pages 746–752

EXAMPLE 2
Find arc lengths

Find the length of $\widehat{AB}$.



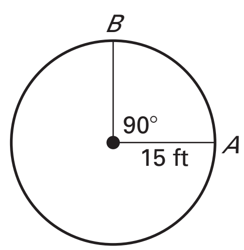
Solution

Arc length of $\widehat{AB} = \frac{150°}{360°} \cdot 2\pi(5) \approx 13.09$ meters.

Exercises for Example 2

Find the length of $\widehat{AB}$.

4.



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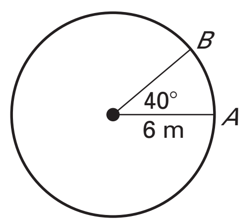

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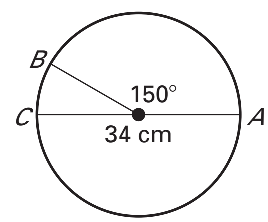
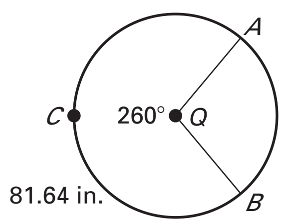


EXAMPLE 3
Use arc lengths to find measures

Find the circumference of $\odot Q$.



Solution

$$\frac{\text{Arc length of } \widehat{ACB}}{C} = \frac{m\widehat{ACB}}{360°}$$

$$\frac{81.64}{C} = \frac{260°}{360°}$$

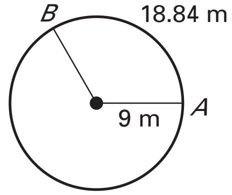
$$113.04 = C$$

The circumference of $\odot Q$ is 113.04 inches.

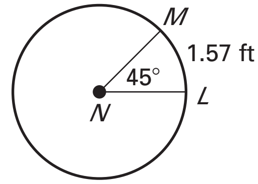
Exercises for Example 3

Find the indicated measure.

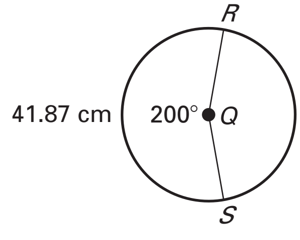
7. $m\widehat{AB}$



8. Radius of $\odot N$



9. Circumference of $\odot Q$



LESSON 11.5
Study Guide
For use with pages 755–761

GOAL **Find the areas of circles and sectors.**

Vocabulary

A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc.

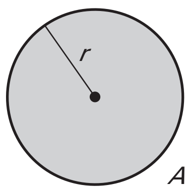
Theorem 11.9 Area of a circle: The area of a circle is π times the square of the radius.

Theorem 11.10 Area of a sector: The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.

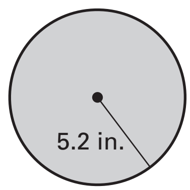
EXAMPLE 1
Use the formula for area of a circle

Find the indicated measure.

a. Radius



b. Area



Solution

a. $A = \pi r^2$ Write the formula for area of a circle.

$201 = \pi r^2$ Substitute 201 for A.

$\frac{201}{\pi} = r^2$ Divide each side by π.

$8 \approx r$ Find the positive square root of each side.

The radius of the circle is about 8 feet.

b. $A = \pi r^2$ Write the formula for area of a circle.

$= \pi \cdot (5.2)^2$ Substitute 5.2 for r.

$= 27.04\pi$ Simplify.

≈ 84.9 Use a calculator.

The area of the circle is about 84.9 square inches.

Exercises for Example 1

Find the indicated measure.

1. The diameter of the circle is 11 centimeters. Find the area.

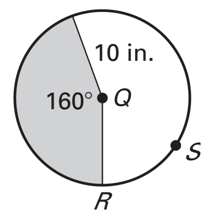
2. The area of the circle is 158.3 square yards. Find the radius.

3. The area of circle is 1024π square meters. Find the diameter.

LESSON **11.5** **Study Guide** *continued*
For use with pages 755–761

EXAMPLE 2 Find the areas of sectors

Find the areas of the sectors formed by ≥ *PQR*.



Solution

STEP 1 **Find** the measures of the minor and major arcs.

Because $m\geq PQR = 160°$, $m\widehat{PR} = 160°$ and $m\widehat{PSR} = 360° - 160° = 200°$.

STEP 2 **Find** the areas of the small and large sectors.

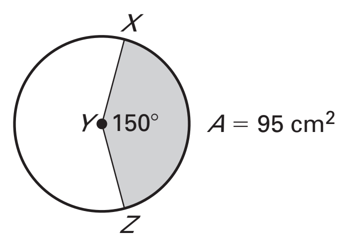
$$\text{Area of small sector} = \frac{160°}{360°} \cdot \pi \cdot 10^2 \approx 139.62$$

$$\text{Area of large sector} = \frac{200°}{360°} \cdot \pi \cdot 10^2 \approx 174.54$$

So, the areas of the small and large sectors are about 139.62 square inches and 174.54 square inches, respectively.

EXAMPLE 3 Use the Area of a Sector Theorem

Use the diagram to find the area of ⊙ *Y*.



Solution

$$\text{Area of sector } XYZ = \frac{m\widehat{XY}}{360°} \cdot \text{Area of } \odot Y$$

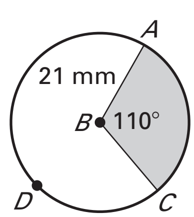
$$95 = \frac{150°}{360°} \cdot \text{Area of } \odot Y$$

$$228 = \text{Area of } \odot Y$$

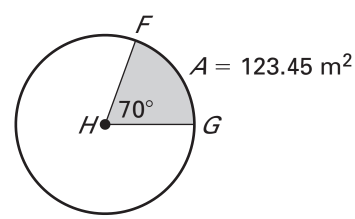
The area of ⊙Y is 228 square centimeters.

Exercises for Examples 2 and 3

4. Find the areas of the sectors formed by ≥ ABC.



5. Find the area of ⊙H.



LESSON **12.4**

Study Guide

For use with pages 819–825

GOAL **Find volumes of prisms and cylinders.**

Vocabulary

The **volume** of a solid is the number of cubic units contained in its interior.

Postulate 27 Volume of a Cube Postulate: The volume of a cube is the cube of the length of its side.

Postulate 28 Volume Congruence Postulate: If two polyhedra are congruent, then they have the same volume.

Postulate 29 Volume Addition Postulate: The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

Theorem 12.6 Volume of a Prism: The volume V of a prism is $V = Bh$ where B is the area of a base and h is the height.

Theorem 12.7 Volume of a Cylinder: The volume V of a cylinder is $V = Bh = \pi r^2 h$, where B is the area of a base, h is the height, and r is the radius of a base.

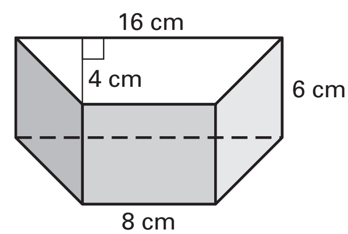
Theorem 12.8 Cavalieri's Principle: If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

EXAMPLE 1

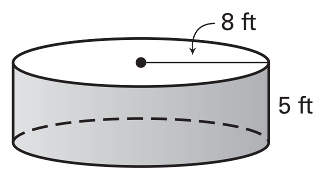
Find volumes of prisms and cylinders

Find the volume of the solid.

a. Right trapezoidal prism



b. Right cylinder



Solution

a. The area of the base is $\frac{1}{2}(4)(16 + 8) = 48 \text{ cm}^2$ and $h = 6$ cm.
$V = Bh = 48(6) = 288 \text{ cm}^3$

b. The area of the base is $\pi r^2 = (8)^2 = 64\pi \text{ ft}^2$. Use $h = 5$ ft to find the volume.
$V = Bh = 64\pi(5) = 320\pi \approx 1005.31 \text{ ft}^3$

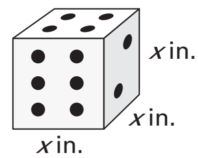
LESSON **12.4**

Study Guide *continued*

For use with pages 819–825

EXAMPLE 2 Use volume of a prism

The volume of the cube is 135 cubic inches. Find the value of *x*.



Solution

A side length of the cube is x inches.

$V = x^3$ Formula for volume of a cube

$135 = x^3$ Substitute for V.

$5.13 \approx x$ Find the cube root.

The value of x is about 5.13 inches.

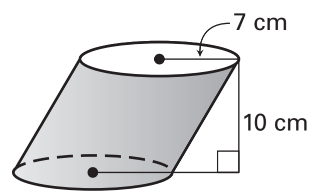
Exercises for Examples 1 and 2

1. Find the volume of a square prism that has a base edge length of 6 feet and a height of 13 feet.

2. The volume of a right cylinder is 896π cubic inches and the height is 14 inches.

EXAMPLE 3 Find the volume of an oblique cylinder

Find the volume of the oblique cylinder.



Solution

Cavalieri's Principle allows you to use Theorem 12.7 to find the volume of the oblique cylinder.

$V = \pi r^2 h$ Formula for volume of a cylinder

$V = \pi(7)^2(10)$ Substitute for known values.

$V = 490\pi$ Simplify.

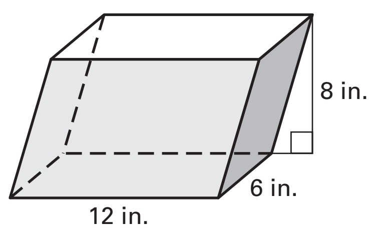
$V \approx 1539.38$ Use a calculator.

The volume of the oblique cylinder is about 1539.38 cubic centimeters.

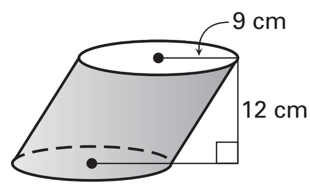
Exercises for Example 3

Find the volume of the oblique prism or cylinder shown.

3.



4.



LESSON 12.5

Study Guide

For use with pages 828–837

GOAL **Volumes of pyramids and cones.**

Vocabulary

Theorem 12.9 Volume of a Pyramid: The volume V of a pyramid is $V = \frac{1}{3}Bh$ where B is the area of the base and h is the height.

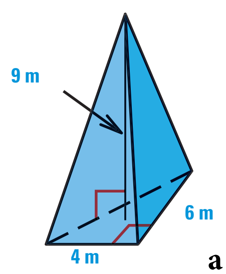
Theorem 12.10 Volume of a Cone: The volume V of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$, where B is the area of the base, h is the height, and r is the radius of the base.

EXAMPLE 1

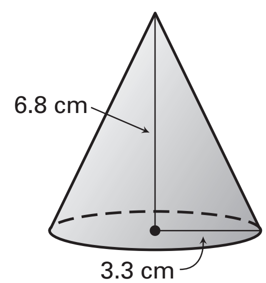
Find the volume of a solid

Find the volume of the solid.

a.



b.



Solution

a. $V = \frac{1}{3}Bh$

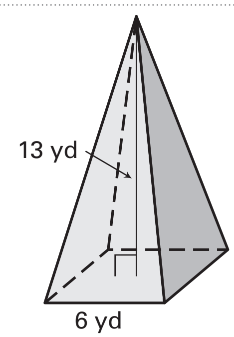
$= \frac{1}{3}\left(\frac{1}{2} \cdot 4 \cdot 6\right)(9) = 36 \text{ m}^3$

b. $V = \frac{1}{3}Bh = \frac{1}{3}(\pi r^2)(h) = \frac{1}{3}(\pi \cdot 3.3^2)(6.8)$

$V = 24.684\pi \approx 77.55 \text{ cm}^3$

Exercise for Example 1

1. Find the volume of the pyramid. Round your answer to two decimal places.



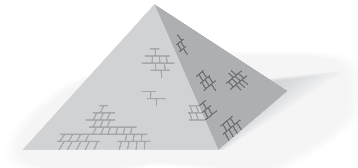
LESSON 12.5

Study Guide *continued*

For use with pages 828–837

EXAMPLE 2 Use volume of a pyramid

The pyramid has a height of 177 meters and volume of 3,465,825 cubic meters. Find the side length of the square base.



Solution

$V = \frac{1}{3}Bh$ Write formula.

$3{,}465{,}825 = \frac{1}{3}(x^2)(177)$ Substitute.

$10{,}397{,}475 = 177x^2$ Multiply each side by 3.

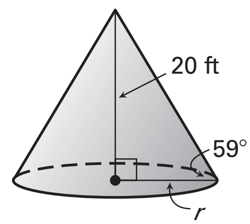
$58{,}743 \approx x^2$ Divide each side by 177.

$242 \approx x$ Find the positive square root.

The side length of the base is about 242 meters.

EXAMPLE 3 Use trigonometry to find the volume of a cone

Find the volume of the right cone.



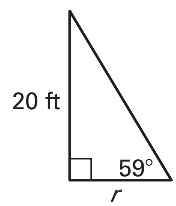
Solution

To find the radius r of the base use trigonometry.

$\tan 59° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio.

$\tan 59° = \frac{20}{r}$ Substitute.

$r = \frac{20}{\tan 59°} \approx 12.02$ Solve for r.



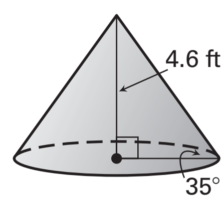
Use the formula for the volume of a cone.

$V = \frac{1}{3}Bh = \frac{1}{3}(\pi r^2)(h) = \frac{1}{3}\pi(12.02^2)(20) \approx 3025.99 \text{ ft}^3$

Exercises for Examples 2 and 3

2. The volume of a right cone is 1275π cubic meters and the radius is 15 meters. Find the height of the cone. Round your answer to two decimal places.

3. Find the volume of the cone at the right. Round your answer to two decimal places.



Graphing Functions

To graph any function by hand, we use a table of values where we pick some x-values and figure out the y-values based on the function given. We do this by substituting the x-value into the function and simplifying.

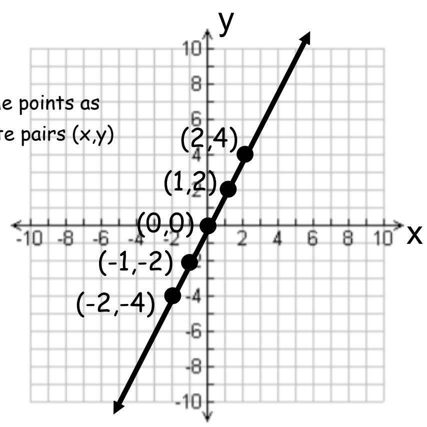
Then, we collect each x and y value as an ordered pair (x,y) and plot them on a graph. The last step is to connect points with a line/curve that fits our points.

Ex) y = 2x

x	y	
-2	2(-2) = -4	→ (-2,-4)
-1	2(-1) = -2	→ (-1,-2)
0	2(0) = 0	→ (0,0)
1	2(1) = 2	→ (1,2)
2	2(2) = 4	→ (2,4)

1. substitute x into function to get a y-value

2. plot the points as coordinate pairs (x,y)

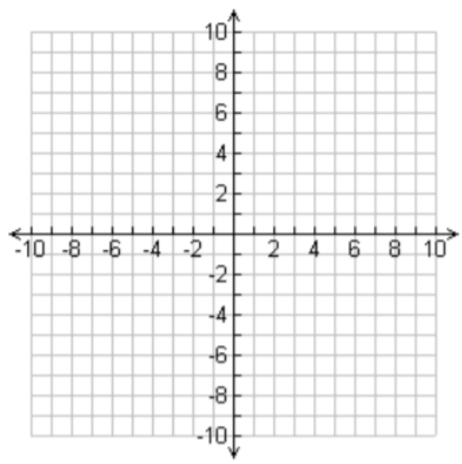


3. connect points with a line/curve that fits the points

You try!

1) y = -2x - 3

x	y
-2	
-1	
0	
1	
2	

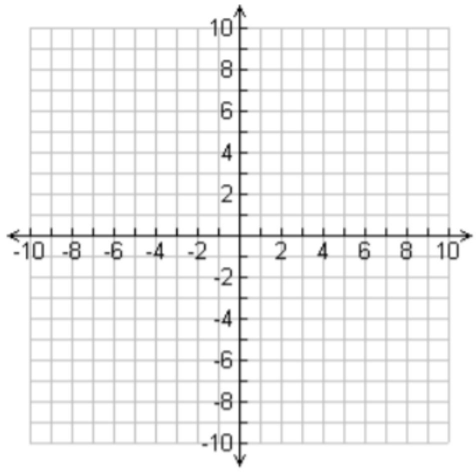


Lesson 21: Graphing

Quadratic

2) $y = x^2$

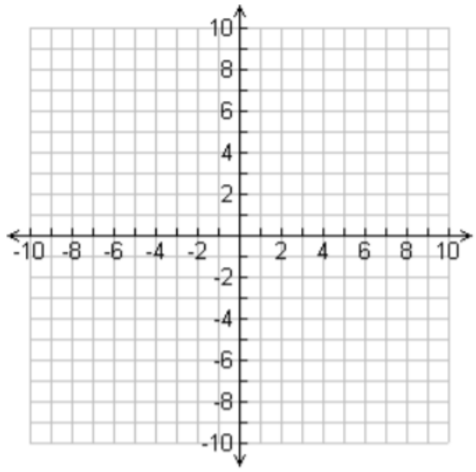
x	y
-2	
-1	
0	
1	
2	



Hint: follow order of operations PEMDAS

3) $y = (x+1)^2 - 5$

x	y
-2	
-1	
0	
1	
2	



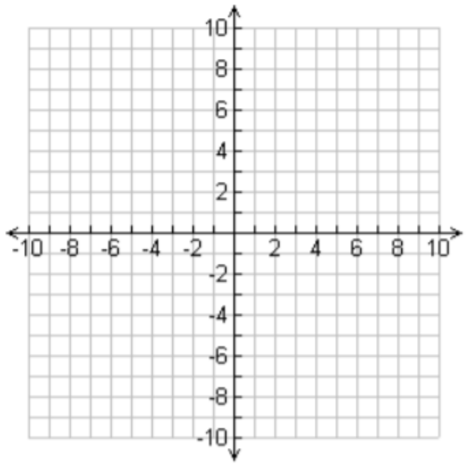
Lesson 21: Graphing

Absolute Value

Hint: absolute value bars describe a distance, which is always positive

4) $y = |x|$

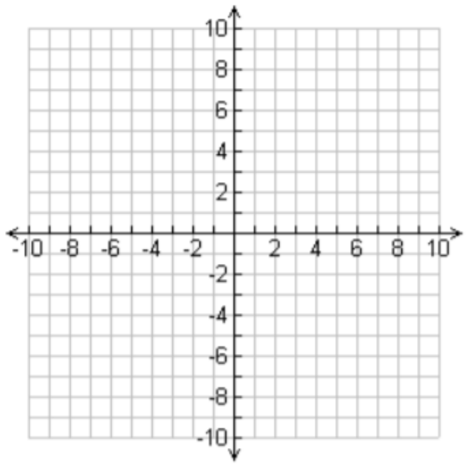
x	y
-2	
-1	
0	
1	
2	



Hint: absolute value bars act like parentheses for order of operations PEMDAS

5) $y = |x + 3| - 6$

x	y
-2	
-1	
0	
1	
2	

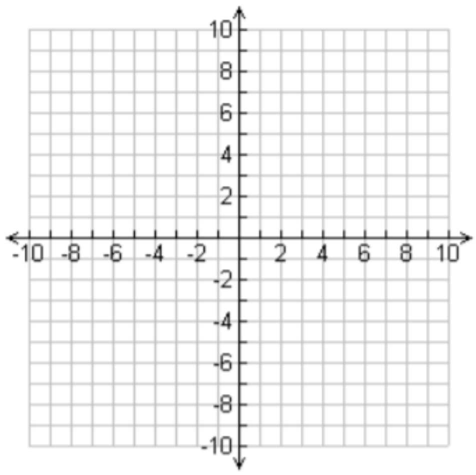


Lesson 21: Graphing

Cubic

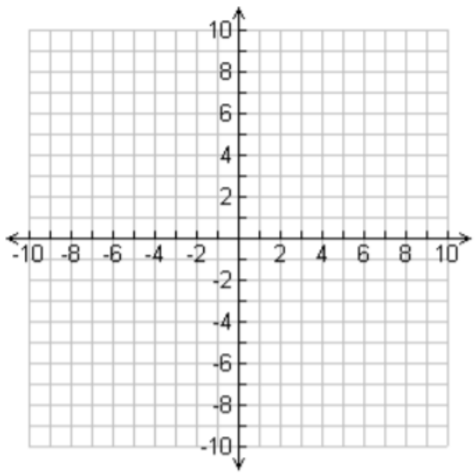
6) $y = x^3$

x	y
-2	
-1	
0	
1	
2	



7) $y = (x-1)^3 - 2$

x	y
-2	
-1	
0	
1	
2	



Lesson 22: Transforming Graphs

Transformations of Functions

We actually could have figured out the graphs of each of those equations without plugging anything into the equation itself. Transformations allow us to look at how changing the equation affects the graph.

$$a \cdot f(bx - h) + k$$

Change to Equation	Change to Graph
f(x) + k	Shift up k units
f(x) - k	Shift down k units
f(x + h)	Shift left h units
f(x - h)	Shift right h units
-f(x)	Reflects across the x axis (flips up or down)
f(-x)	Reflects across the y axis (flips left or right)
af(x)	a>1 vertical stretch
	0<a<1 vertical compression
f(bx)	b>1 horizontal compression
	0<b<1 horizontal stretch

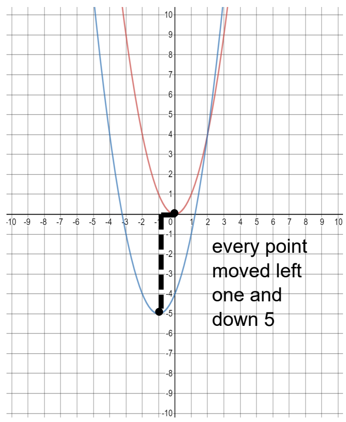
Lets look at the quadratics from the previous assignment

Parent Function: $y = x^2$

$$y = (x+1)^2 - 5$$

+1 is on the inside of the ( ) so it is our h value. This shifts the graph **left** 1 unit.

-5 is on the outside of the ( ) so it is our k value. This shifts the graph **down** 5 units



Example #1

$$y = 2|x| + 3$$

- 3 is added on the outside (k)
- So this a **shift up 3**

- 2 is multiplied on the outside (a)
- 2 is greater than 1
- So this is a **vertical stretch by a factor of 2**

Example #2

$$y = -(\tfrac{1}{4}x)^3$$

- negative is on the outside
- so this is a **reflection across the x axis**

- 1/4 is on the inside (b)
- 1/4 is between 0 and 1
- so this is a **horizontal stretch by a factor 4**

Parent Graphs & Transformations

For problem 1- 9, please give the name of the parent function and describe the transformation represented.

Parent Functions

Linear	Absolute Value	Quadratic	Cubic	Square root
$y = x$	$y = \|x\|$	$y = x^2$	$y = x^3$	$y = \sqrt{x}$

1. $g(x) = x^2 - 1$ Parent: ____

Transformations: __________________________________

2. $f(x) = 2|x-1|$ Parent: ____

Transformations: __________________________________

3. $h(x) = -\sqrt{2x} - 2$ Parent: ____

Transformations: __________________________________

4. $g(x) = -2(x+1)^2 + 3$ Parent: ____

Transformations: __________________________________

5. $g(x) = -3x - 2$ Parent: ____

Transformations: __________________________________

6. $f(x) = |x+5| - 2$ Parent: ____

Transformations: __________________________________

7. $h(x) = (-x-2)^3 + 4$ Parent: ____

Transformations: __________________________________

Lesson 22: Transforming Graphs

8. $h(x) = -x^2 + 1$ Parent: ____________________________________

Transformations:______________________________

9. $h(x) = -|x - 2|$ Parent: ____________________________________

Transformations:______________________________

For problems 10 – 13, given the parent function and a description of the transformation, write the equation of the transformed function, f(x).

10. Absolute value—vertical shift up 5, horizontal shift right 3. _________________________________

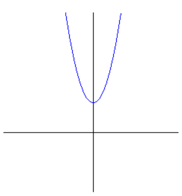
11. Linear—vertical compression by $\frac{2}{5}$ ____

12. Square Root —reflected over the x axis, vertical shift down 2.________________________________

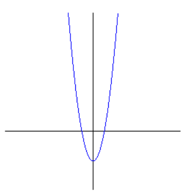
13. Quadratic—vertical stretch by 5, horizontal shift left 8.______________________________________

14. Which graph best represents the function $f(x) = 2x^2 - 2$?

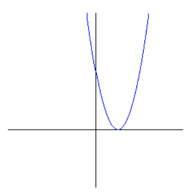
a.



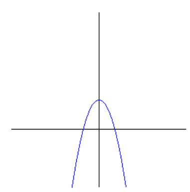
b.



c.



d.



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Analyzing and Interpreting Scientific Data

How can analyzing and interpreting scientific data allow scientists to make informed decisions?

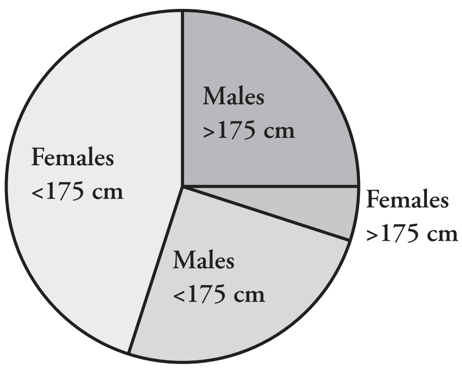
Why?

During scientific investigations, scientists gather data and present it in the form of charts, tables or graphs. The data must be properly collected, analyzed, and interpreted to allow scientists to make informed decisions regarding the validity of their study and any further work that may be necessary to achieve their objectives. The ability to present and use data charts, tables, and graphs correctly is essential for good scientific practice and also prevents unnecessary or inappropriate work and misinterpretation of the data.

Model 1 – Graphs and Charts of Classroom Measurement Data

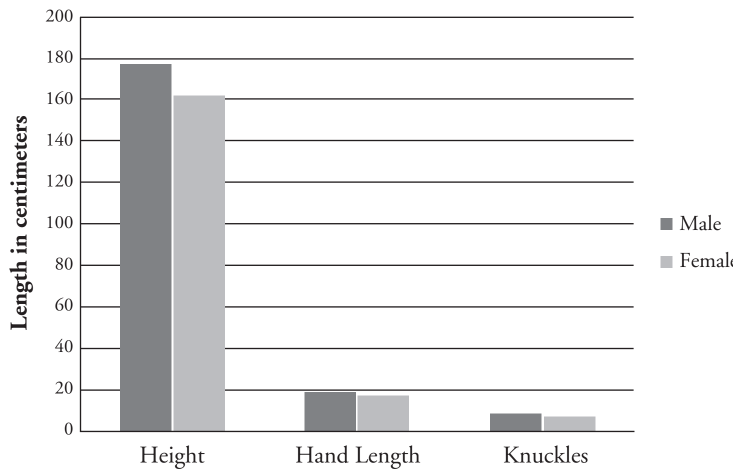
Pie Chart

Percentage of Males and Females by Height



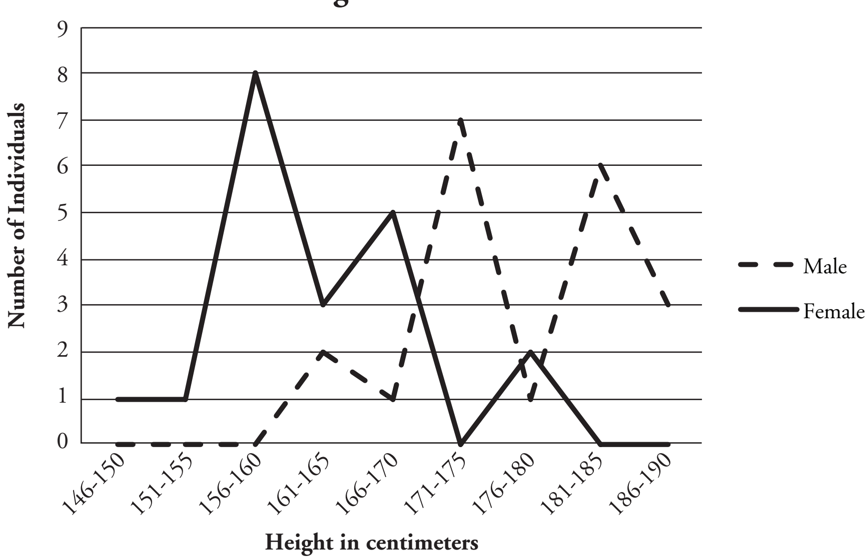
Bar Graph

Comparing Male and Female Average Values



Line Graph

Distribution of Height in Males and Females



1. According to the data in Model 1, how many females fall within the range 146–155 cm tall?

2. According to the data in Model 1, how many males are 181 cm or above in height?

3. Using the graph(s) in Model 1, determine the approximate average height of males and of females.

4. Refer to the data in Model 1.

   a. How many males are taller than 175 cm and approximately what percentage of the total is that?

   b. Which graph(s)/chart(s) illustrate the answer to the previous question?

5. Which type of graph or chart in Model 1 shows a side by side comparison of data?

6. Which type of graph or chart in Model 1 shows trends in data across an entire data set?

7. Describe two trends in male and female height using the line graph.

8. Use complete sentences to compare the presentation of height data in the three graphs. Discuss any information that is located on more than one graph, and any unique information that is available on each.

9. If you wanted to see if a correlation exists between the height of an individual and his/her hand length, what would be the best type of graph/chart to make? Explain your reasoning.

10. What conclusions can you draw comparing the height, hand length, and knuckle width of males and females? State your conclusions in complete sentences.

STOP

Model 2 – Foot Width in a High School Classroom

Female foot width (cm)	Male foot width (cm)
7.8	10
8	10.5
8	9
5	9.3
17	13
7.5	7.5
7.5	10
7	9.2
7.8	9
7	4.5

$$\textbf{Mean} = \frac{\text{sum of all data values}}{\text{number of data values}}$$

Median = Middle value of an ordered set of data.

Mode = Most frequently occurring value in a set of data.

11. Refer to the data in Model 2.
    a. What value for foot width is most frequent in males?

    b. What is this value called?

12. Determine the median value for foot width for males and for females. Describe in complete sentences the method you used to determine the median values.

13. Determine the mean for each data group, and describe in a complete sentence how you calculated them.

STOP

Read This!

Within a data set there may be individual values that seem uncharacteristic or do not fit the general trend. These data points may be referred to as **outliers** or **anomalous data**. In most samples, a small number of outliers is to be expected, due to the variation inherent in any naturally-occurring population. Outliers can also result from errors in measurement or in the recording of data. Normal variation can often be distinguished from error by repeating the measurements to see if the same range is obtained. Scientists also use statistical calculations to determine the expected range of data, so that judgments can be made about the authenticity of individual data points. Outliers should not be ignored, however, as many interesting scientific discoveries have resulted from the study of such unexpected findings.

14. Which data point(s) in the foot width values in Model 2 might be considered outliers? Explain your choice(s).

15. The equation below allows you to calculate the amount of deviation (in percent) for the values within a data set. The percent deviation is reported as an absolute value.

$$\% \text{ deviation} = \frac{|(\text{mean value using all data}) - (\text{mean value excluding anomalous data})|}{\text{mean value using all data}} \times 100$$

a. What is the percent deviation in the female data set when the outlying value of 17 is excluded (*i.e.,* considered to be anomalous data)?

$$\% \text{ deviation} = \frac{|8.26 - 7.29|}{8.26} \times 100 = 11.7\%$$

b. What is the percent deviation in the male data set when the outlying value of 4.5 is excluded?

$$\% \text{ deviation} = \frac{|9.20 - 9.72|}{9.20} \times 100 = 5.65\%$$

c. Which data set (male or female) had the largest percent deviation?

16. Given the outliers and amount of deviation in each data set, which value (mean, median, mode) *best represents* the overall data set of foot width in males and females? Explain your answer in a complete sentence.

STOP

Extension Questions

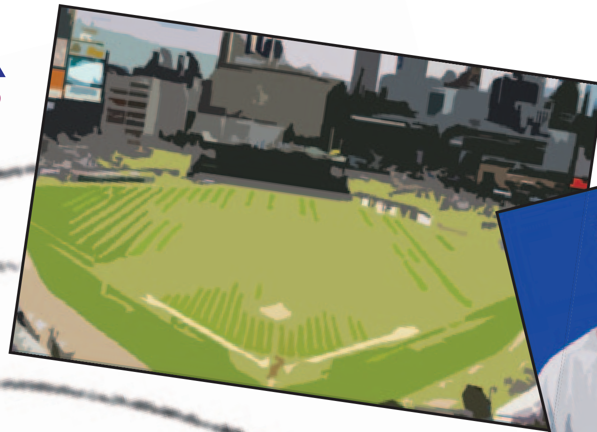
17. With your group, discuss the issues below relating to data analysis and scientific ethics, and record your answers in complete sentences.

    a. What could you do to determine whether the outliers in Model 2 are authentic measurements?

    b. Under which circumstances would it be appropriate to remove outlying data points from the analysis and conclusions in a scientific study?

    c. If you were to decide to remove outlying data points from your analysis, what are two ways you could indicate this in your report to ensure you are being honest about your data analysis?

Student Explorations in Mathematics



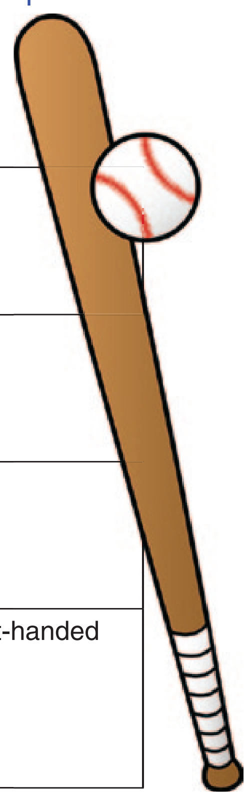
Out of the Park! Exploring the Mean in Sports

In 2012, Detroit Tiger Miguel Cabrera became the first baseball player since 1967 to win the American League Triple Crown, leading the league in batting average (.330), home runs (44), and runs batted in—RBIs—(139). During the 2012 season, Cabrera had 50 hits in 159 times at bat versus left-handed pitchers and 155 hits in 463 times at bat versus right-handed pitchers. In Cabrera's rookie season (2003), he had 20 hits in 55 times at bat versus left-handed pitchers and 64 hits in 259 times at bat versus right-handed pitchers.

Part 1. Comparing Batting Averages in Baseball

Using the information in the introduction, calculate Cabrera's batting averages below:

1. a. In 2012, versus left-handed pitchers	b. In 2012, versus right-handed pitchers
2. a. In 2003, versus left-handed pitchers	b. In 2003, versus right-handed pitchers
3. a. Overall batting average, 2012	b. Overall batting average, 2003
4. a. Combined batting average versus left-handed pitchers for the years 2003 and 2012	b. Combined batting average versus right-handed pitchers for the years 2003 and 2012



5. Was Cabrera's batting average better during the 2012 or 2003 season? How did you reach this conclusion?

Part 2. Means in Basketball

Britney has averaged 14.1 points per game for her team's first 10 basketball games. Her goal was to average 15.0 points per game, but only 1 game remains.

6. Britney believes that if she scores 18 points in her final game, she will achieve her goal. Do you agree? Why or why not?

7. If you do not agree, what is the minimum number of points she must score in her final game to achieve her goal? How did you arrive at your conclusion?

Part 3. Grade Point Averages (GPA)

Chris is concerned about his GPA. In five of his six classes, he has an A, B, A–, B+, and an A–, respectively. Below are the corresponding values for each letter grade.

A	4.00
A–	3.67
B+	3.33
B	3.00
B–	2.67
C+	2.33
C	2.00
C–	1.67
D+	1.33
D	1.00
D–	0.67
F	0.00

8. If a GPA of 3.2 will put Chris on the honor roll, what grade could he have in his sixth class to achieve this goal?

9. If a GPA of 3.5 will earn Chris an academic letter, what must his grade be in his sixth class?

Part 4. Comparing Means of Data Sets

10. Ms. Issippi teaches two geography classes. She gives the same 50-point test to both classes. One class averages 41 points. The other averages 45 points. How is it possible that the combined test average is not 43?

Mrs. Johnson teaches two advanced algebra classes. She gives the same 100-point test to both classes, with one class averaging 78 points, and the other, 82 points. One class has 25 students, and the other class has 20 students, but it is not known which is which.

11. What is the highest average possible for the two classes combined? What reasoning did you use to find your answer?

12. What is the lowest average possible for the two classes combined?

13. Six students took the test at a later time and averaged 76. Provide a sample of six scores that would yield an average of 76 for the six students.

14. Suppose five students took the test at a later time and averaged 79. Provide a sample of five scores that would yield an average of 79 for the five students.

A **stem plot**, sometimes called a stem-and-leaf plot, is often used for smaller quantities of data (in a data set, 30–50 individual values are ideal for a stem plot). Stem plots are useful for showing the distribution within a data set and can also be used to compare two data sets side by side, as shown on the next page. In the stem plot on the next page, the numbers between the vertical lines represent the tens place, and the numbers to the left or right of the vertical lines represent the units digits. The data are arranged in order, allowing the user to easily find the minimum, maximum, and range. Other measures that can be read easily from a stem plot are the mean, median, mode, and quartiles. Below are methods for calculating the median, mode, and quartiles.

Median (also known as the second quartile, or Q2): If a set of data has an odd number of values, the median is calculated by finding the middle value after all values have been arranged in order. If a set of data has an even number of values, the median is calculated by finding the mean of the two middle values. This measure of center divides the data into two equal halves.

First Quartile (Q1): The first quartile is the median of the set of values less than the median and is calculated in the same way as the median, but using only those values less than the median.

Third Quartile (Q3): The third quartile is the median of the set of values greater than the median and is calculated in the same way as the median, but using only values greater than the median.

Range: Maximum value – minimum value

Mode: Most frequent value

Lesson 24: Out of the Park - Using Data Sets

Use the following stem plot of the two advanced algebra classes' test scores to answer the questions that follow.

Algebra Test Scores

1st period						**5th period**							
				0	10	0	0						
	1	4	6	7	9	1	3	6	8	9			
	2	3	7	9	8	2	4	6	7	8	8		
1	1	3	3	7	7	1	2	4	4	6	9	9	9
0	2	4	4	7	6	6	8	8					
				9	5	2							

15. Determine the median for each class. Which class has the mean of 78, and which one has 82? Which is higher in each class, the mean or the median? Why?

16. What is the mode for the first period? What is the mode for the fifth period?

17. What is the range for each class?

18. Calculate the first and third quartiles for each class.

19. Construct a box plot using the data from the stem plot above.

20. Which class average would be affected more if those students who scored 100 had their scores removed?

Can you . . .

- determine what influences the mean and median?
- construct box plots from data in a different graphical display?
- simplify fractions with expressions in the numerator and denominator?
- create algebraic equations to solve problems involving means?
- count how many ways you use mean every day?

LESSON 13.1

Study Guide

For use with pages 842–848

GOAL **Find sample spaces and probabilities.**

Vocabulary

A possible result of an experiment is an **outcome.**

An **event** is an outcome or a collection of outcomes, such as rolling an odd number.

The set of all possible outcomes is called a **sample space.**

The **probability of an event** is a measure of the likelihood, or chance, that the event will occur.

The **odds** of an event compare the number of favorable and unfavorable outcomes when all outcomes are equally likely.

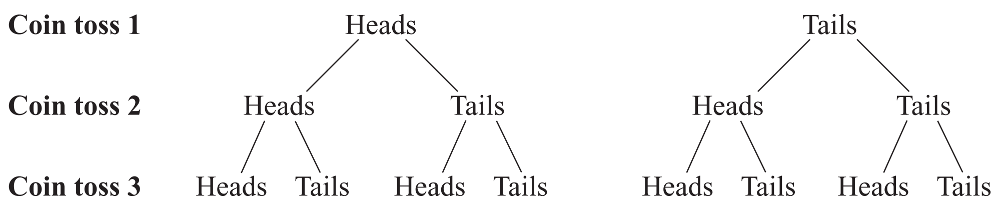
EXAMPLE 1

Find a sample space

You toss 3 coins. How many possible outcomes are in the sample space? List the possible outcomes.

Solution

Use a tree diagram to find the outcomes in the sample space.



The sample space has 8 possible outcomes. They are listed below. (Heads, H; Tails, T)

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

Exercise for Example 1

1. A spinner has 5 congruent spaces numbered 1 through 5. You spin the spinner and toss a coin. Find the number of possible outcomes. Then list the possible outcomes.

EXAMPLE 2 Find a theoretical probability

A bag contains numbered balls in red, blue, and yellow. The table below shows the numbers of each type of ball. A ball is selected at random. What is the probability that the ball selected is an odd numbered yellow ball?

	Red	Blue	Yellow
Even numbered	6	8	10
Odd numbered	11	7	8

Solution

There is a total of $6 + 8 + 10 + 11 + 7 + 8 = 50$ balls. So, there are 50 possible outcomes. Of all the balls, 8 are odd numbered and yellow. There are 8 possible favorable outcomes.

$$
\begin{aligned}
P(\text{odd and yellow}) &= \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} \\
&= \frac{\text{Number of odd, yellow balls}}{\text{Total number of balls}} \\
&= \frac{8}{50} \\
&= \frac{4}{25}
\end{aligned}
$$

Exercises for Example 2

In Exercises 2–4, use the table from Example 2 to find the probability.

2. What is the probability that a randomly chosen ball is even and red?

3. What is the probability that a randomly chosen ball is *not* odd and blue?

4. What is the probability that a randomly chosen ball is odd?

LESSON 13.4

Study Guide

For use with pages 861–867

GOAL **Find the probability of compound events.**

Vocabulary

A **compound event** combines two or more events, using the word *and* or the word *or*.

Mutually exclusive events have no common outcomes.

Overlapping events have at least one common outcome.

Two events are **independent events** if the occurrence of one event has no effect on the occurrence of the other.

Two events are **dependent events** if the occurrence of one event affects the occurrence of the other.

EXAMPLE 1 Find the probability of A or B

You randomly choose a card from a standard 52-card deck.

a. Find the probability that you choose a Queen or an Ace.

b. Find the probability that you choose a King or a club.

Solution

a. Choosing a Queen or an Ace are mutually exclusive events.

$$P(\text{Queen } or \text{ Ace}) = P(\text{Queen}) + P(\text{Ace})$$
$$= \frac{4}{52} + \frac{4}{52}$$
$$= \frac{8}{52} = \frac{2}{13}$$

b. Because there is a King of clubs, choosing a King and a club are overlapping events. There are 4 Kings, 13 clubs, and 1 card that is both.

$$P(\text{King } or \text{ club}) = P(\text{King}) + P(\text{club}) - P(\text{King and club})$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$
$$= \frac{16}{52} = \frac{4}{13}$$

Exercises for Example 1

1. You choose a card from a standard 52-card deck. Find the probability that you choose a heart or a spade.

2. You choose a card from a standard 52-card deck. Find the probability that you choose an even number or a heart.

EXAMPLE 2 **Find the probability of A and B**

Inspection A quality inspector at a screw manufacturer randomly selects one screw from each batch of 50 screws to inspect for problems or non-conformance. The first batch of 50 screws has 4 non-conforming screws. The second batch of 50 screws has 5 non-conforming screws. Find the probability that the inspector selects a non-conforming screw both times.

Solution

The events are independent. The selection from the first batch of screws does not affect the selection from the second batch of screws.

$P(\text{non-conforming in batch 1}) = \frac{4}{50}$ $\quad$ $P(\text{non-conforming in batch 2}) = \frac{5}{50}$

Multiply the probabilities of the two events:

$P(\text{both screws non-conforming}) = \frac{4}{50} \cdot \frac{5}{50} = \frac{1}{125}$

The probability of randomly choosing non-conforming screws both times is $\frac{1}{125}$.

EXAMPLE 3 **Find the probability of A and B**

Goldfish An aquarium contains 6 male goldfish and 4 female goldfish. You randomly select a fish from the tank, do not replace it, and then randomly choose a second fish. What is the probability that both fish are male?

Solution

Because you do not replace the first fish, the events are dependent. Before choosing the first fish there are 10 fish, 6 of them male. After the first male fish is chosen, there are 9 fish and 5 of them are male.

$$P(\text{male and then male}) = P(\text{male}) \cdot P(\text{male given male})$$
$$= \frac{6}{10} \cdot \frac{5}{9} = \frac{1}{3}$$

Exercise for Examples 2 and 3

3. **Apples** A basket of apples contains 6 red apples, 2 green apples, and 3 yellow apples. You randomly select 2 apples, one at a time. Find the probability that both are yellow if:

a. you replace the first apple, then select the second.

b. you eat the first apple, then select the second.

LESSON 13.2

Study Guide

For use with pages 851–855

GOAL **Use the formula for the number of permutations.**

Vocabulary

A **permutation** is an arrangement of objects in which order is important.

For any positive integer n, the product of the integers from 1 to n is called ***n* factorial** and is written as $n!$.

EXAMPLE 1 Count permutations

Consider the number of permutations of the letters in the word APRIL.

a. In how many ways can you arrange all of the letters?

b. In how many ways can you arrange 3 of the letters?

Solution

a. Use the counting principle to find the number of permutations of the letters in the word APRIL.

Number of Permutations	=	Choices for 1st letter	•	Choices for 2nd letter	•	Choices for 3rd letter	•	Choices for 4th letter	•	Choices for 5th letter
	=	5	•	4	•	3	•	2	•	1
	= 120									

There are 120 ways you can arrange all of the letters in the word APRIL.

b. When arranging 3 letters of the word APRIL, you have 5 choices for the first letter, 4 for the second letter, and 3 for the third letter.

Number of Permutations	=	Choices for 1st letter	•	Choices for 2nd letter	•	Choices for 3rd letter
	=	5	•	4	•	3
	= 60					

There are 60 ways you can arrange 3 of the letters in the word APRIL.

Exercises for Example 1

Count the permutations.

1. In how many ways can you arrange the letters in the word FLOWER?

2. In how many ways can you arrange 4 of the letters in the word PANTHER?

3. In how many ways can you arrange 2 of the letters in the word COMPUTER?

13.2 **Study Guide** *continued*
For use with pages 851–855

EXAMPLE 2 Use permutation formula

Packing You have 11 pairs of shorts and plan to pack 5 of them for a vacation. In how many ways can you choose the shorts you pack for your vacation?

Solution

To find the number of permutations of 5 pairs of shorts chosen from 11, find ${}_{11}P_5$.

$${}_{11}P_5 = \frac{11!}{(11-5)!}$$ Permutation formula

$$= \frac{11!}{6!}$$ Subtract.

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}}$$ Expand factorials. Divide out common factorial, 6!.

$$= 55{,}440$$ Multiply.

There are 55,440 ways to arrange 5 pairs of shorts out of 11.

EXAMPLE 3 Find a probability using permutations

Softball There are 10 players on a softball team. Each game the batting order is randomly fixed. Find the probability that you are chosen to bat first, and your best friend is chosen to bat second.

Solution

STEP 1 **Write** the number of possible outcomes as the number of permutations of the 10 players on the team. This is ${}_{10}P_{10} = 10!$.

STEP 2 **Write** the number of favorable outcomes as the number of permutations of the players given that you are the first batter, and your best friend is the second. This is ${}_8P_8 = 8!$.

STEP 3 **Calculate** the probability.

$$P\begin{pmatrix}\text{You are first batter,} \\ \text{Best friend is second batter}\end{pmatrix} = \frac{8!}{10!}$$ Form a ratio of favorable to possible outcomes.

$$= \frac{\cancel{8!}}{10 \cdot 9 \cdot \cancel{8!}}$$ Expand factorial. Divide out common factor, 8!.

$$= \frac{1}{90}$$ Simplify.

Exercises for Examples 2 and 3

4. **What if?** In Example 2, suppose you have 9 pairs of shorts. You pack 4 of them for your vacation. In how many ways can you choose the shorts you pack for vacation?

5. **What if?** In Example 3, suppose there are 15 players on the team and that everyone gets to bat. Find the probability that you are the first batter and that your best friend is the second batter.

LESSON 13.3

Study Guide

For use with pages 856–860

GOAL **Use combinations to count possibilities.**

Vocabulary

A **combination** is a selection of objects in which order is *not* important.

EXAMPLE 1

Count combinations

Count the combinations of three letters from the list A, B, C, D.

Solution

List all of the permutations of three letters from the list A, B, C, D. Because order is not important in a combination, cross out any duplicate groupings.

ABC, ~~ACB~~, ABD, ~~ADB~~, ACD, ~~ADC~~

~~BAC~~, ~~BCA~~, BCD, ~~BDC~~, ~~BDA~~, ~~BAD~~

~~CAB~~, ~~CBA~~, ~~CBD~~, ~~CDB~~, ~~CAD~~, ~~CDA~~

~~DAB~~, ~~DBA~~, ~~DAC~~, ~~DCA~~, ~~DBC~~, ~~DCB~~

There are 4 possible combinations of 3 letters from the list A, B, C, D.

Exercise for Example 1

1. Count the combinations of 2 letters from the list A, B, C, D, E, F.

EXAMPLE 2

Use combination formula

Photo Background For your school picture, you can choose 4 backgrounds from a list of 10. How many combinations of backdrops are possible?

Solution

The order in which you choose the backgrounds is not important. So, to find the number of combinations of 10 backgrounds taken 4 at a time, find ${}_{10}C_4$.

$${}_{10}C_4 = \frac{10!}{(10-4)! \cdot 4!}$$ Combination formula

$$= \frac{10!}{6! \cdot 4!}$$ Subtract.

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot (4 \cdot 3 \cdot 2 \cdot 1)}$$ Expand factorials. Divide out common factorial, 6!.

$$= 210$$ Multiply.

There are 210 different combinations of backgrounds you can choose.

LESSON 13.3 **Study Guide** *continued*
For use with pages 856–860

EXAMPLE 3 Find a probability using combinations

Student Council A school's student council has 16 members, including 4 seniors. There are 4 members randomly chosen to represent the student council at a school open house. What is the probability that all 4 council members chosen are the seniors?

Solution

STEP 1 **Write** the number of possible outcomes as the number of combinations of 16 people chosen 4 at a time, or ${}_{16}C_4$, because the order in which the people are chosen is not important.

$$
\begin{aligned}
{}_{16}C_4 &= \frac{16!}{(16-4)! \cdot 4!} \\
&= \frac{16!}{12! \cdot 4!} \\
&= \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot \cancel{12!}}{\cancel{12!} \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\
&= 1820
\end{aligned}
$$

STEP 2 **Find** the number of favorable outcomes. Only one of the possible outcomes includes all four seniors.

STEP 3 **Calculate** the probability.

$$P(\text{all seniors are chosen}) = \frac{1}{1820}$$

Exercises for Examples 2 and 3

2. **What if?** In Example 2, suppose you can choose 3 backgrounds out of the list of 10. How many combinations are possible?

3. **What if?** In Example 3, suppose there are 12 members on student council, 4 of them seniors. Find the probability the seniors are the 4 members chosen for the open house.

Lesson 27: Domain and Range

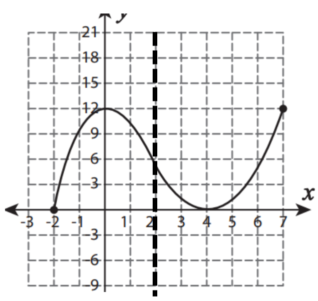
Domain and Range from Graphs

Function: A relation is a function if for every input there is exactly one output.

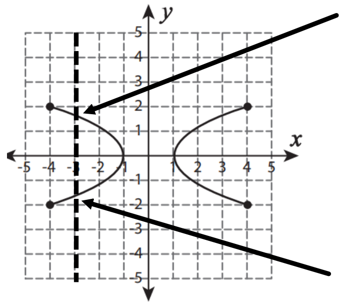
This means we can use a vertical line to test if a graph is a function.

The dashed lines would be drawn in by you

ex: This is a function



ex: This is NOT a function

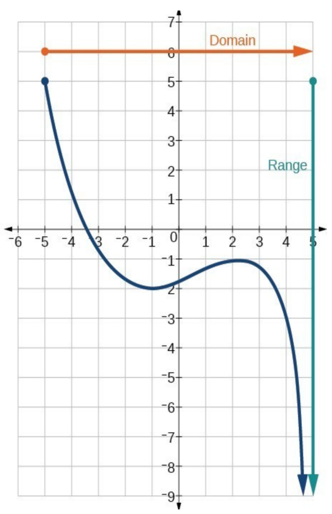


If you can draw any vertical line that touches the graph twice, its not a function

The **domain** refers to the set of possible input values, which are shown on the x-axis.

The **range** refers to the set of possible output values, which are shown on the y-axis.

Example #1



Use brackets when the dot is solid, this means that point is a part of the domain and range

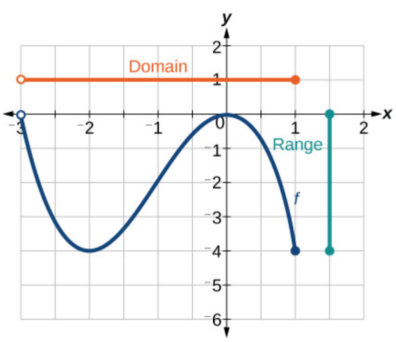
Domain: [-5, ∞)

Range: [5, -∞)

Use a parenthesis when you have a dot that is not filled in or an arrow

reminder: an arrow means the graph is going to continue on off towards infinity

Example #2



Note: this is parenthesis because there is a hole in the graph at -3

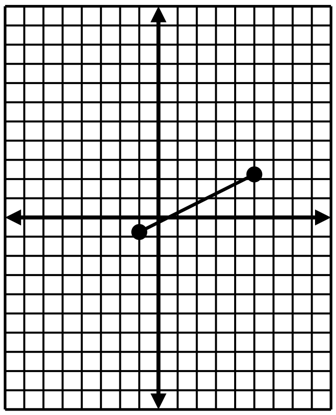
Domain: (-3, 1]

Range: [-4, 0]

Lesson 27: Domain and Range

For each problem: a) State the domain
b) State the range
c) Determine if the graph is a function

1.

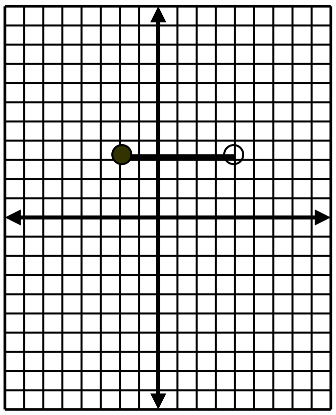


a)________________

b)________________

c) ________________

2.

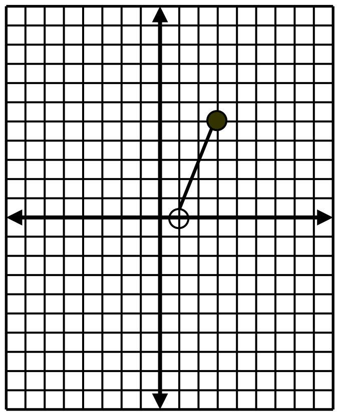


a)________________

b)________________

c)________________

3.

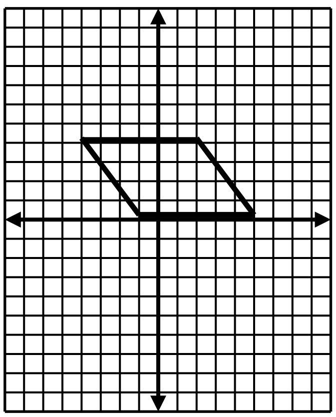


a)________________

b)________________

c)________________

4.

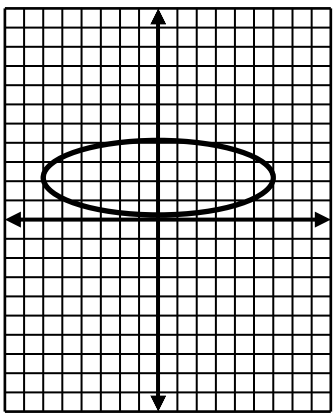


a)________________

b)________________

c) ________________

5.

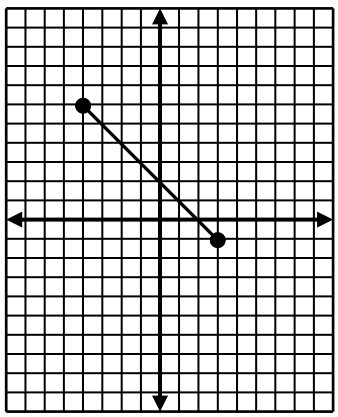


a)________________

b)________________

c)________________

6.



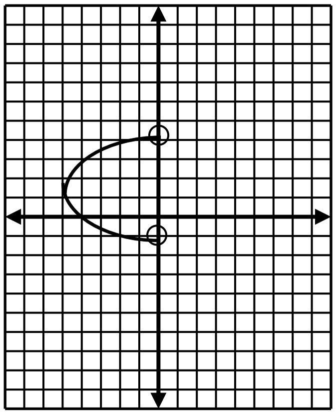
a)________________

b)________________

c)________________

Lesson 27: Domain and Range

7.

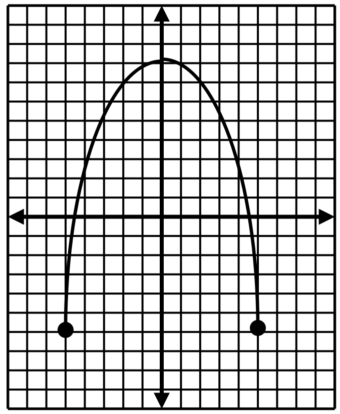


a)________________

b)________________

c) ________________

8.

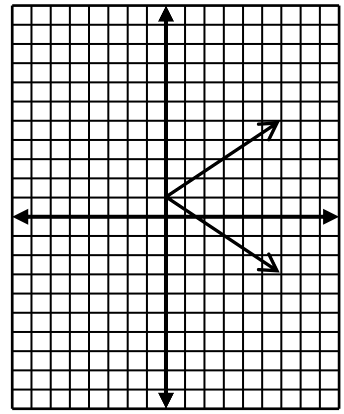


a)________________

b)________________

c)________________

9.

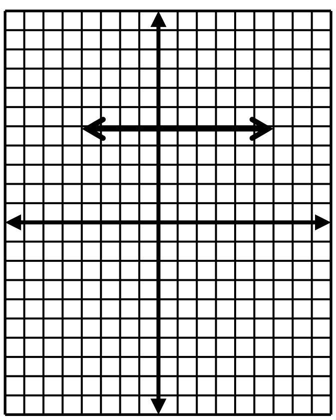


a)________________

b)________________

c)________________

10.

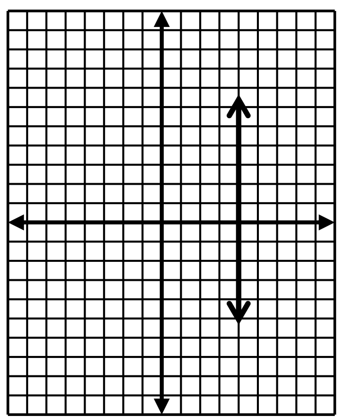


a)________________

b)________________

c) ________________

11.

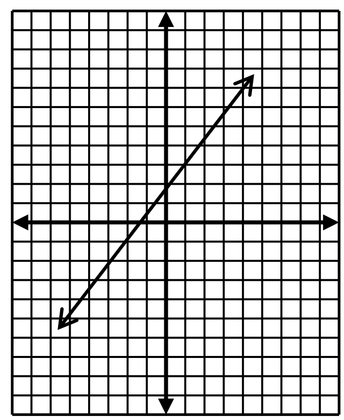


a)________________

b)________________

c)________________

12.



a)________________

b)________________

c)________________

13. Tara's car travels about 25 miles on one gallon of gas. She has between 10 and 12 gallons of gas in the tank.
   a) List the independent and dependent quantities.

   b) Find the reasonable domain and range values.

   c) Write the reasonable domain and range as inequalities.

14. Sal and three friends plan to bowl one or two games each. Each game costs $2.50.
   a) List the independent and dependent quantities.

   b) Find the reasonable domain and range values.

   c) Write the reasonable domain and range as inequalities.

Lesson 28: Composite Functions

Composite Functions

First, a bit of review.

f(x) is what we call function notation. It is extremely useful because it names the function and tells what the input is. This allows us to have multiple functions in the same problem (i.e. g(x), h(x), etc.).

f(x) represents the output. x represents the input. When x is replaced with a number [like f(3)] that means 3 will replace all of the x's in the expression that represents the function.

if $f(x) = x^2 - 4x + 2$

then $f(3) = 3^2 - 4 \bullet 3 + 2$

the input has been defined to be 3, so we put a 3 into each spot (x) we need an input value.

This same relationship holds even if our input is something weird, like a blue scribble.

$$f(\text{ʕ}) = \text{ʕ}^2 - 4 \bullet \text{ʕ} + 2$$

once we define the input for the function to be a blue scribble, that blue scribble is put in for each x (or whatever variable the input was)

Composite Functions

Since we make most anything the input for a function (even a blue scribble!) it makes sense that we could also input a function

let $f(x) = x^2 - 4x + 2$ and $g(w) = 2^w$

The input variable really doesn't matter. I'm using the two different variable to emphasize the different functions and inputs

If we want to use g as the input for f, we can.

so, $f(g(w))$ means we put g(w) each time we have an x in f

$$f(g(w)) = (g(w))^2 - 4g(w) + 2$$

but since we have an expression for g(w)

$$f(g(w)) = (2^w)^2 - 4 \bullet 2^w + 2$$

The order in which we compose functions does matter.

if we wanted g(f(x)), we would be using f as the input for g. This means that f(x) would replace each w in g

$$g(f(x)) = 2^{x^2 - 4x = 2}$$

note that this is a very different function than f(g(w)) (besides just having a different input variable)

We do not need to have a two named functions to have a composite. For example:

$$f(\sin\theta) = (\sin\theta)^2 - 4\sin\theta + 2$$

since sine is a function, this is also a composite

Lesson 28: Composite Functions

GOAL **Perform operations with functions.**

Vocabulary

A **power function** has the form $y = ax^b$ where a is a real number and b is a rational number.

The **composition** of a function g with a function f is: $h(x) = g(f(x))$ where the domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.

EXAMPLE 1

Add and subtract functions

Let $f(x) = -3x^{1/3}$ and $g(x) = -x^{1/3}$. Find the following.

a. $f(x) + g(x)$ **b.** $f(x) - g(x)$ **c.** the domains of $f + g$ and $f - g$

Solution

a. $f(x) + g(x) = -3x^{1/3} + \left(-x^{1/3}\right) = [(-3) + (-1)]x^{1/3} = -4x^{1/3}$

b. $f(x) - g(x) = -3x^{1/3} - \left(-x^{1/3}\right) = [(-3) - (-1)]x^{1/3} = -2x^{1/3}$

c. The functions f and g each have the same domain: all real numbers. So, the domains of $f + g$ and $f - g$ also consist of all real numbers.

EXAMPLE 2

Multiply and divide functions

Let $f(x) = 2x^2$ and $g(x) = x^{1/2}$. Find the following.

a. $f(x) \cdot g(x)$ **b.** $\dfrac{f(x)}{g(x)}$ **c.** the domains of $f \cdot g$ and $\dfrac{f}{g}$

Solution

a. $f(x) \cdot g(x) = \left(2x^2\right)\left(x^{1/2}\right) = 2x^{(2\,+\,1/2)} = 2x^{5/2}$

b. $\dfrac{f(x)}{g(x)} = \dfrac{2x^2}{x^{1/2}} = 2x^{(2\,-\,1/2)} = 2x^{3/2}$ for x greater than zero.

c. The domain of f consists of all real numbers, and the domain of g consists of all nonnegative real numbers. So the domain of $f \cdot g$ consists of all nonnegative real numbers. Because $g(0) = 0$, the domain of $\dfrac{f}{g}$ is restricted to all positive real numbers.

Exercises for Examples 1 and 2

Let $f(x) = 2x^2$ and $g(x) = -3x^2$. Perform the indicated operation. State the domain.

1. $f(x) + g(x)$ **2.** $f(x) - g(x)$ **3.** $f(x) \cdot g(x)$ **4.** $\dfrac{f(x)}{g(x)}$

LESSON 6.3 **Study Guide** *continued*
For use with pages 428–435

EXAMPLE 3 **Find compositions of functions**

Let $f(x) = 2x^3$ and $g(x) = x^{-1}$. Find the following.

a. $f(g(x))$ **b.** $g(f(x))$ **c.** the domain of each composition

Solution

a. $f(g(x)) = f(x^{-1}) = 2(x^{-1})^3 = 2x^{-3} = \frac{2}{x^3}$

b. $g(f(x)) = g(2x^3) = (2x^3)^{-1} = \frac{1}{2x^3}$

c. The domain of $f(g(x))$ consists of all real numbers except 0 because $x = 0$ is not in the domain of g. The domain of $g(f(x))$ consists of all real numbers except 0 because $f(0) = 0$ is not in the domain of g.

EXAMPLE 4 **Solve a multi-step problem**

Sports Store You purchase water skis with a price tag of $180 dollars. The sports store applies a newspaper coupon of $50 and a 10% store discount. Use composition to find the final price of the purchase when the coupon is applied before the discount. Use composition to find the final price of the purchase when the discount is applied before the coupon.

STEP 1 **Write** functions for the discounts. Let x be the tag price, $f(x)$ be the price after the $50 coupon, and $g(x)$ be the price after the 10% store discount.

Function for $50 coupon: $f(x) = x - 50$

Function for 10% discount: $g(x) = x - 0.10x = 0.90x$

STEP 2 **Compose** the functions.

$50 coupon is applied first: $g(f(x)) = g(x - 50) = 0.90(x - 50)$

10% discount is applied first: $f(g(x)) = f(0.90x) = 0.90x - 50$

STEP 3 **Evaluate** the functions $g(f(x))$ and $f(g(x))$ when $x = 180$.

$g(f(180) = 0.90(180 - 50) = 0.90(130) = \$117$

$f(g(180) = 0.90(180) - 50) = 162 - 50 = \$112$

The final price is $117 when the $50 coupon is applied before the 10% discount. The final price is $112 when the 10% discount is applied before the $50 coupon.

Exercises for Examples 3 and 4

5. Let $f(x) = 3x^2$ and $g(x) = x^2 + 5$. Find (a) $f(g(x))$ and (b) $g(f(x))$.

6. Rework Example 4 for a price tag of $200, a $30 coupon, and a 20% discount.

Lesson 29: SAT Practice - Composite Functions

- Reasoning with more complex equations and functions in one variable
- Understanding and solving questions using the graphs of quadratic and other higher-order functions

Include proof of work that led to your solution.

1) If $f(x)$ is defined as $f(x) \ll x^2 + 1$ and $g(x)$ is defined as $g(x) \ll x - 1$, what is the rule for $h(x) \ll f(g(x))$?

A) $h(x) \ll x^2 + 2x + 2$

B) $h(x) \ll x^2 + 2$

C) $h(x) \ll x^2 - 2$

D) $h(x) \ll x^2 - 2x + 2$

2) The equation for $g(x)$ is $g(x) \ll x^2$. The function $h(x)$ is the function $g(x)$ moved 3 units to the right. The function $f(x)$ is the function $h(x)$ stretched by a factor of 3. What is the equation for $f(x)$?

A) $f(x) \ll 3(x+3)^2$

B) $f(x) \ll 3(x-3)^2$

C) $f(x) \ll 3x^2 + 3$

D) $f(x) \ll 3x^2 - 3$

3) The equation for $g(x)$ is $g(x) \ll |x+2|$. The function $h(x)$ is the function $g(x)$ compressed by a factor of 0.5. The function $f(x)$ is the function $h(x)$ shifted 2 units to the right. What is the equation for $f(x)$?

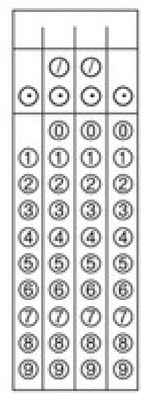
A) $f(x) \ll |0.5x + 2|$

B) $f(x) \ll |x+2| - 0.5$

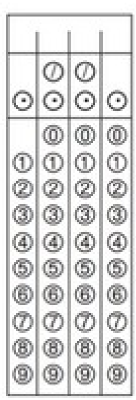
C) $f(x) \ll 0.5|x|$

D) $f(x) \ll |x| - 0.5$

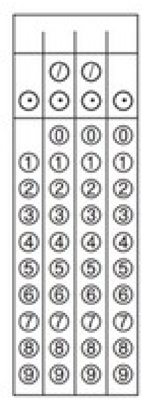
4) If $f(x) \ll |x| + 3$ and $g(x) \ll \sqrt{x+1}$. Find the value of $g(f(-5))$



5) If $f(x) \ll \sqrt{x-11}$ and $g(x) \ll x^2$. Find the value of $f(g(6))$



6) If $f(x) \ll \frac{x-5}{2}$ and $g(x) \ll \sqrt{3x^2 + 4}$. Find the value of $g(f(9))$



Lesson 29: SAT Practice - Composite Functions

⁀ Reasoning with more complex equations and functions in one variable
⁀ Understanding and solving questions using the graphs of quadratic and other higher-order functions

Include proof of work that led to your solution

1) Let $p(x) \ll x - 2$ and $q(x) \ll \sqrt{x}$. If $f(x) \ll q(p(x))$, find $f(6)$.

A) 2

B) ~ 2

C) 4

D) $-2 + \sqrt{6}$

2) If $f(x)$ is defined as $f(x) \ll x + 4$ and $g(x)$ is defined as $g(x) \ll x^2 - 2$, what is the rule for $h(x) \ll f(g(x))$?

A) $h(x) \ll x^2 + 8x + 14$

B) $h(x) \ll x^2 + 8x + 18$

C) $h(x) \ll x^2$

D) $h(x) \ll x^2 + 2$

3) The equation for $g(x)$ is $g(x) \ll |x|$. The function $h(x)$ is the function $g(x)$ compressed by a factor of 0.5. The function $f(x)$ is the function $h(x)$ moved 2 units to the left. What is the equation for $f(x)$?

A) $f(x) \ll |0.5x| + 2$

B) $f(x) \ll |0.5x| - 2$

C) $f(x) \ll 0.5|x + 2|$

D) $f(x) \ll 0.5|x - 2|$

4) If $m(x) \ll 2 - 4x$ and $k(x) \ll x^2 - 3x$, find $m(k(-1))$.

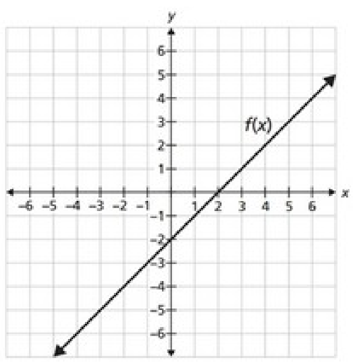
A) 18

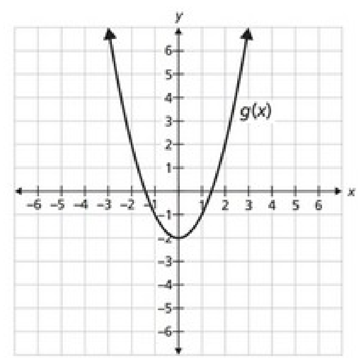
B) 14

C) -14

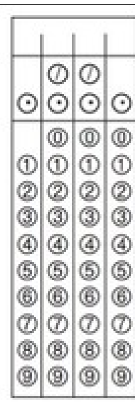
D) -18

Questions 5 and 6 refer to the graphs of f(x) and g(x) below.

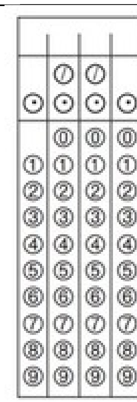




5) Evaluate $h(x) \ll g(f(0))$.



6) Evaluate $h(x) \ll f(g(-2))$.



Linear Equations - Word Problems

Word problems can be tricky. Often it takes a bit of practice to convert the english sentence into a mathematical sentence. This is what we will focus on here with some basic number problems, geometry problems, and parts problems.

A few important phrases are described below that can give us clues for how to set up a problem.

- **A number** (or unknown, a value, etc) often becomes our variable
- **Is** (or other forms of is: was, will be, are, etc) often represents equals (=)

  x is 5 becomes $x = 5$

- **More than** often represents addition and is usually built backwards, writing the second part plus the first

  Three more than a number becomes $x + 3$

- **Less than** often represents subtraction and is usually built backwards as well, writing the second part minus the first

  Four less than a number becomes $x - 4$

Using these key phrases we can take a number problem and set up and equation and solve.

Example 1.

If 28 less than five times a certain number is 232. What is the number?

$5x - 28$ Subtraction is built backwards, multiply the unknown by 5

$5x - 28 = 232$ Is translates to equals

then solve

Example 2.

Fifteen more than three times a number is the same as ten less than six times the number. What is the number

$3x + 15$ First, addition is built backwards

$6x - 10$ Then, subtraction is also built backwards

$3x + 15 = 6x - 10$ Is between the parts tells us they must be equal

then solve

Another type of number problem involves consecutive numbers. **Consecutive numbers** are numbers that come one after the other, such as 3, 4, 5. If we are looking for several consecutive numbers it is important to first identify what they look like with variables before we set up the equation. This is shown in Example 3.

Example 3.

The sum of three consecutive integers is 93. What are the integers?

$$\begin{array}{rl} \text{First } x & \text{Make the first number } x \\ \text{Second } x+1 & \text{To get the next number we go up one or } +1 \\ \text{Third } x+2 & \text{Add another 1 (2 total) to get the third} \\ F+S+T=93 & \text{First } (F) \text{ plus Second } (S) \text{ plus Third } (T) \text{ equals 93} \\ (x)+(x+1)+(x+2)=93 & \text{Replace } F \text{ with } x, S \text{ with } x+1, \text{ and } T \text{ with } x+2 \end{array}$$

then solve

Sometimes we will work consective even or odd integers, rather than just consecutive integers. When we had consecutive integers, we only had to add 1 to get to the next number so we had x, $x + 1$, and $x + 2$ for our first, second, and third number respectivly. With even or odd numbers they are spaced apart by two. So if we want three consecutive even numbers, if the first is x, the next number would be $x + 2$, then finally add two more to get the third, $x + 4$. The same is true for consecutive odd numbers, if the first is x, the next will be $x + 2$, and the third would be $x + 4$. It is important to note that we are still adding 2 and 4 even when the numbers are odd. This is because the phrase "odd" is refering to our x, not to what is added to the numbers. Consider the next two examples.

Example 4.

The sum of three consecutive even numbers is 246. What are the numbers?

$$\begin{array}{rl} \text{First } x & \text{Make the first } x \\ \text{Second } x+2 & \text{Even numbers, so we add 2 to get the next} \\ \text{Third } x+4 & \text{Add 2 more (4 total) to get the third} \\ F+S+T=246 & \text{Sum means add First } (F) \text{ plus Second } (S) \text{ plus Third } (T) \\ (x)+(x+2)+(x+4)=246 & \text{Replace each } F, S, \text{ and } T \text{ with what we labeled them} \end{array}$$

then solve

Example 5.

Find three consecutive odd integers so that the sum of twice the first, the second and three times the third is 152.

$$\begin{array}{rl} \text{First } x & \text{Make the first } x \\ \text{Second } x+2 & \text{Odd numbers so we add 2 (same as even!)} \\ \text{Third } x+4 & \text{Add 2 more (4 total) to get the third} \\ 2F+S+3T=152 & \text{Twice the first gives } 2F \text{ and three times the third gives } 3T \\ 2(x)+(x+2)+3(x+4)=152 & \text{Replace } F, S, \text{ and } T \text{ with what we labled them} \end{array}$$

then solve

When we started with our first, second, and third numbers for both even and odd we had x, $x + 2$, and $x + 4$. The numbers added do not change with odd or even, it is our answer for x that will be odd or even.

Another example of translating english sentences to mathematical sentences comes from geometry. A well known property of triangles is that all three angles will always add to 180. For example, the first angle may be 50 degrees, the second 30 degrees, and the third 100 degrees. If you add these together, $50 + 30 + 100 = 180$. We can use this property to find angles of triangles.

Example 6.

The second angle of a triangle is double the first. The third angle is 40 less than the first. Find the three angles.

First x	With nothing given about the first we make that x
Second $2x$	The second is double the first,
Third $x - 40$	The third is 40 less than the first
$F + S + T = 180$	All three angles add to 180
$(x) + (2x) + (x - 40) = 180$	Replace F, S, and T with the labeled values.

then solve

Another geometry problem involves perimeter or the distance around an object. For example, consider a rectangle has a length of 8 and a width of 3. Their are two lengths and two widths in a rectangle (opposite sides) so we add $8 + 8 + 3 + 3 = 22$. As there are two lengths and two widths in a rectangle an alternative to find the perimeter of a rectangle is to use the formula $P = 2L + 2W$. So for the rectangle of length 8 and width 3 the formula would give, $P = 2(8) + 2(3) = 16 + 6 = 22$. With problems that we will consider here the formula $P = 2L + 2W$ will be used.

Example 7.

The perimeter of a rectangle is 44. The length is 5 less than double the width. Find the dimensions.

Length x	We will make the length x
Width $2x - 5$	Width is five less than two times the length
$P = 2L + 2W$	The formula for perimeter of a rectangle
$(44) = 2(x) + 2(2x - 5)$	Replace P, L, and W with labeled values

then solve

Lesson 30: Write Equations From Context (Story Problems)

We have seen that it is imortant to start by clearly labeling the variables in a short list before we begin to solve the problem. This is important in all word problems involving variables, not just consective numbers or geometry problems. This is shown in the following example.

Example 8.

A sofa and a love seat together costs $444. The sofa costs double the love seat. How much do they each cost?

Love Seat x	With no information about the love seat, this is our x
Sofa $2x$	Sofa is double the love seat, so we multiply by 2
$S + L = 444$	Together they cost 444, so we add.
$(x) + (2x) = 444$	Replace S and L with labled values
	then solve

Be careful on problems such as these. Many students see the phrase "double" and believe that means we only have to divide the 444 by 2 and get $222 for one or both of the prices. As you can see this will not work. By clearly labeling the variables in the original list we know exactly how to set up and solve these problems.

Practice - Word Problems

Solve.

1. When five is added to three more than a certain number, the result is 19. What is the number?

2. If five is subtracted from three times a certain number, the result is 10. What is the number?

3. A number plus itself, plus twice itself, plus 4 times itself, is equal to -104. What is the number?

4. Sixty more than nine times a number is the same as two less than ten times the number. What is the number?

5. Fourteen less than eight times a number is three more than four times the number. What is the number?

6. The sum of three consecutive integers is 108. What are the integers?

7. The sum of three consecutive integers is -126. What are the integers?

8. The sum of three consecutive odd integers is 189. What are the integers?

14. The second angle of a triangle is the same size as the first angle. The third angle is 12 degrees larger than the first angle. How large are the angles?

15. Two angles of a triangle are the same size. The third angle is 12 degrees smaller than the first angle. Find the measure the angles.

16. The perimeter of a rectangle is 150 cm. The length is 15 cm greater than the width. Find the dimensions.

17. The perimeter of a rectangle is 152 meters. The width is 22 meters less than the length. Find the length and width.

30. A mountain cabin on 1 acre of land costs 30,000 dollars. If the land cost 4 times as much as the cabin, what was the cost of each?

31. A bicycle and a bicycle helmet cost 240 dollars. How much did each cost, if the bicycle cost 5 times as much as the helmet?

32. Of 240 stamps that harry and his sister collected, Harry collected 3 times as many as his sisters. How many did each collect?

33. Aaron had 7 times as many sheep as Beth, and both together had 608. How many sheep had each?

34. A man bought a cow and a calf for 990 dollars, paying 8 times as much for the cow as for the calf. what was the cost of each?

35. An electrician cuts a 30 ft piece of wire into two pieces. One piece is 2 ft longer than the other. How long are the pieces?

Lesson 31: SAT Practice: Write Equations From Context (Story Problems)

L3-1

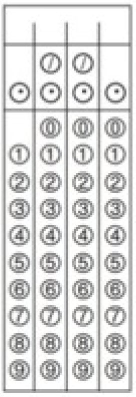
SAT Problems of the Week - Heart of Algebra
Creating, Evaluating, and Interpreting Linear Functions

Include proof of work that led to your solution.

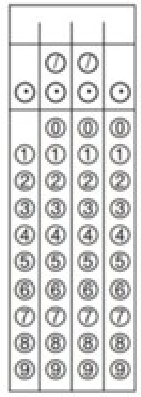
1) **Questions 1 – 3 refer to the following information.**

The function f is defined as $f(x) = 2x + 5$. Find the value for each of the following:

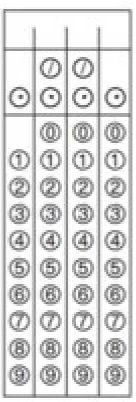
$f(7) =$



2) $f(0) =$



3) $f(-2.5) =$



4) A garden has room for 5 rows of flowers. There are four additional flowers, one in each corner of the arrangement. Write a function to describe the total number of flowers in the garden plot if each row has x flowers.

A) $f(x) = (5 + 4)x$
B) $f(x) = 5x + 4$
C) $f(x) = 6x - 4$
D) $f(x) = 5x + 20$

5) It costs a certain amount of money to prepare a book for sale. Suppose the linear function that represents the total revenue gained from the sale of the book is $f(n) = 3n - 3{,}000$, where n represents the number of copies sold. Which of the following is the best interpretation of the function $f(n)$?

A) It cost \$3.00 per copy to prepare each book for sale and the company earned \$3,000 from the book sales.
B) The company spent \$3,000 preparing the book for sale, which cost \$3.00 per copy, so they sold 100 books.
C) The book is sold for \$3.00 per copy and the company gains a profit of \$3,000 from the sales of the book.
D) The company invested \$3,000 in preparing the book for printing. The difference between the sale price of each copy and the cost to print each copy is \$3.00.

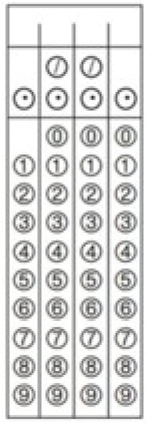
Answers: 1. (19) 2. (5) 3. (0) 4. (B) 5. (D)

Lesson 31: SAT Practice: Write Equations From Context (Story Problems)

L1-2 Formulas and Absolute Value/Representing Relationships Using Algebraic Language/Finding Equivalent Expression

Include proof of work that led to your solution. .

1) The formula for the area of a kite with diagonals d_1 and d_2 is $A = \frac{d_1 \cdot d_2}{2}$. Find the area of a kite with diagonals of lengths $\sqrt{2}$ and $\sqrt{8}$.



2) Which of the following expressions has the same value as $\frac{|-6|}{-2} - \frac{|4-7|}{6}$?

A) $\frac{-2|-3|}{3} + 4|3-1|$

B) $\frac{-|9-5|}{8} + \frac{|1-7|}{2}$

C) $\frac{-|2-4|}{4} + \frac{-3|2-5|}{3}$

D) $\frac{|-9|}{3} - \frac{|2-8|}{3}$

3) The difference between the square root of a number n and the number n squared.

A) $\sqrt{n} - n$

B) $\sqrt{n - n^2}$

C) $\sqrt{n} - n^2$

D) $n^2 - \sqrt{n}$

4) The sum of a and b decreased by their product.

A) $ab - (a - b)$

B) $a + b - ab$

C) $a + b + ab$

D) $a + b - \frac{a}{b}$

5) The product of m and n divided by three times their difference.

A) $mn - 3(m - n)$

B) $\frac{mn}{3(m-n)}$

C) $\frac{m}{n} - 3(m - n)$

D) $\frac{m+n}{3(m-n)}$

6) Three times the difference of a and b divided by twice their sum.

A) $\frac{3(a-b)}{2(a+b)}$

B) $\frac{2(a+b)}{3(a-b)}$

C) $3(a-b) - 2(a+b)$

D) $\frac{3a-b}{2a+b}$

7) $m^2n - 4m^2 + 3mn^2 - (-m^2n + 3mn^2 - 2m^2)$

Which of the following is equivalent to the expression above?

A) $7m^2 + mn^2$

B) $2m^2n - 6m^2$

C) $2m^2n - 2m^2$

D) $2m^2n - 2m^2 + 6mn^2$

8) $\frac{3x^2y + 6xy - 9xy^2}{3}$

Which of the following is equivalent to the expression above?

A) $x^2y + 2xy + 3xy^2$

B) $-2x^2y + 2xy$

C) $-2x^2y - 2xy$

D) $x^2y + 2xy - 3xy^2$

Lesson 32: Write Systems From Context (Story Problems)

Example:

1) You are selling tickets for a high school play. Student tickets cost \$4 and general admission tickets cost \$6. You sell 525 tickets and collect \$2876. How many of each type of ticket did you sell?

Step 1: Define Variables

The things we don't know are the number of each type of ticket

Let S = the number of student tickets

and G = the number of general admission tickets

Step 2: Write Equations that relate those variables

We know the total number of tickets is the sum of student and general

$$S + G = 525$$

We know that the amount of money we get for each type of ticket is the cost times the number of tickets and we know the total amount of money we get

$$4*S + 6*G = 2676$$

Step 3: Solve the system like any other to get the answer

2) You are ordering softballs for two softball leagues. The Pony league uses an 11-inch softball priced at \$2.75. The Junior League uses a 12-inch softball priced at \$3.25. The bill smeared in the rain, but you know that total was 80 softballs for \$245. How many of each size did you order?

3) Your math teacher tells you that next week's test is worth 100 points and contains 38 problems. Each problem is worth either 5 points or 2 points. Because you are studying systems of linear equations, your teachers says that for extra credit you can figure out how many problems of each value are on the test. How many of each value are there?

Lesson 32: Write Systems From Context (Story Problems)

4) A wilderness group is selling cans of nuts and boxes of microwaveable popcorn to raise money for a trip. A can of nuts sells for $4.50 and a box of microwaveable popcorn sells for $3. The group sells $252 in nuts and popcorn and they sells twice as many boxes of popcorn as cans of nut. How many boxes of popcorn and cans of nuts did the group sell?

6) You are planning a birthday party for your cousin. You can have the party at a pizza place for $8 per person plus $30 for party favors and a small cake, or you can have the party at a taco place for $12 per person plus $14 for a large cake. How many children would you have to invite to the party for the cost to be the same for both places? What will the final cost of having the party at either place be?

7) You plant a 14-inch hemlock tree in your backyard that grows at a rate of 4 inches per year and an 8-inch blue spruce tree that grows at a rate of 6 inches per year. In how many years after you plant the trees will the two trees be the same height? How tall will the trees be?

8) Last year you mowed and shoveled snow for 10 households. You earned $200 per household mowing for the entire season and $180 per household shoveling for the entire season. If you earned a total of $1880 last year, how many households did you mow and shovel for?

9) You have $1 bills and $5 bills in your wallet. You have 15 bills in all. You count the money and find that you have $47. How many of each type of bill do you have in your wallet?

Lesson 33: SAT Practice - Write Systems From Context (Story Problems)

11

$$b = 2.35 + 0.25x$$
$$c = 1.75 + 0.40x$$

In the equations above, b and c represent the price per pound, in dollars, of beef and chicken, respectively, x weeks after July 1 during last summer. What was the price per pound of beef when it was equal to the price per pound of chicken?

A) \$2.60
B) \$2.85
C) \$2.95
D) \$3.35

6

An online bookstore sells novels and magazines. Each novel sells for \$4, and each magazine sells for \$1. If Sadie purchased a total of 11 novels and magazines that have a combined selling price of \$20, how many novels did she purchase?

A) 2
B) 3
C) 4
D) 5

19

A food truck sells salads for \$6.50 each and drinks for \$2.00 each. The food truck's revenue from selling a total of 209 salads and drinks in one day was \$836.50. How many salads were sold that day?

A) 77
B) 93
C) 99
D) 105

10

Between 1497 and 1500, Amerigo Vespucci embarked on two voyages to the New World. According to Vespucci's letters, the first voyage lasted 43 days longer than the second voyage, and the two voyages combined lasted a total of 1,003 days. How many days did the second voyage last?

A) 460
B) 480
C) 520
D) 540

10

A group of 202 people went on an overnight camping trip, taking 60 tents with them. Some of the tents held 2 people each, and the rest held 4 people each. Assuming all the tents were filled to capacity and every person got to sleep in a tent, exactly how many of the tents were 2-person tents?

A) 30

B) 20

C) 19

D) 18

11

A software company is selling a new game in a standard edition and a collector's edition. The box for the standard edition has a volume of 20 cubic inches, and the box for the collector's edition has a volume of 30 cubic inches. The company receives an order for 75 copies of the game, and the total volume of the order to be shipped is 1,870 cubic inches. Which of the following systems of equations can be used to determine the number of standard edition games, s, and collector's edition games, c, that were ordered?

A) $75 - s = c$
$20s + 30c = 1{,}870$

B) $75 - s = c$
$30s + 20c = 1{,}870$

C) $s - c = 75$
$25(s + c) = 1{,}870$

D) $s - c = 75$
$30s + 20c = 1{,}870$

9

A worker uses a forklift to move boxes that weigh either 40 pounds or 65 pounds each. Let x be the number of 40-pound boxes and y be the number of 65-pound boxes. The forklift can carry up to either 45 boxes or a weight of 2,400 pounds. Which of the following systems of inequalities represents this relationship?

A) $\begin{cases} 40x + 65y \le 2{,}400 \\ x + y \le 45 \end{cases}$

B) $\begin{cases} \dfrac{x}{40} + \dfrac{y}{65} \le 2{,}400 \\ x + y \le 45 \end{cases}$

C) $\begin{cases} 40x + 65y \le 45 \\ x + y \le 2{,}400 \end{cases}$

D) $\begin{cases} x + y \le 2{,}400 \\ 40x + 65y \le 2{,}400 \end{cases}$

6

Two types of tickets were sold for a concert held at an amphitheater. Tickets to sit on a bench during the concert cost $75 each, and tickets to sit on the lawn during the concert cost $40 each. Organizers of the concert announced that 350 tickets had been sold and that $19,250 had been raised through ticket sales alone. Which of the following systems of equations could be used to find the number of tickets for bench seats, B, and the number of tickets for lawn seats, L, that were sold for the concert?

A) $(75B)(40L) = 1{,}950$
$B + L = 350$

B) $40B + 75L = 19{,}250$
$B + L = 350$

C) $75B + 40L = 350$
$B + L = 19{,}250$

D) $75B + 40L = 19{,}250$
$B + L = 350$

Lesson 34: SAT Practice - General Algebra

SAT Problems of the Week - Heart of Algebra
L1-1 Simplifying and Evaluating Algebraic Expressions

Include proof of work that led to your solution.

1) Simplify $2x^3 - 4x + 8 - x^3 + 5x - 15$.

A) $x^3 + x - 7$
B) $3x^3 + x - 15$
C) $x^3 - x - 7$
D) $x^3 + x + 15$

2) Simplify $-2(3y - 1) + y + 1$.

A) $-6y - 1$
B) $-5y - 1$
C) $-6y + 3$
D) $-5y + 3$

3) Evaluate $(a + b)^2(a - c)$ when $a = -1$, $b = -2$, and $c = 1$.

A) -4
B) -6
C) -9
D) -18

4) Evaluate $a - c(b + c)$ when $a = 1$, $b = 2$, and $c = -1$.

A) 2
B) 1
C) -1
D) -2

5) In the figure below, segment $\overline{PR}$ is divided into 2 segments. The length of $\overline{PQ}$ is $x + y$, and the length of $\overline{QR}$ is $x - y$. If $\overline{PQ}$ is twice as long as $\overline{QR}$ and $x = 15$, then what is the length of $\overline{PR}$?

Note: Figure not drawn to scale.

A) 5
B) 20
C) 25
D) 30

6) If $2c = -1$, then $(4 - 4c)^2 =$

A) 6
B) 36
C) 4
D) 2

7) What is the result when $10x - 7$ is subtracted from $9x - 15$?

A) $x + 8$
B) $x + 22$
C) $-x - 8$
D) $-x + 8$

8) What does $5(x + y) - (5x - y)$ equal?

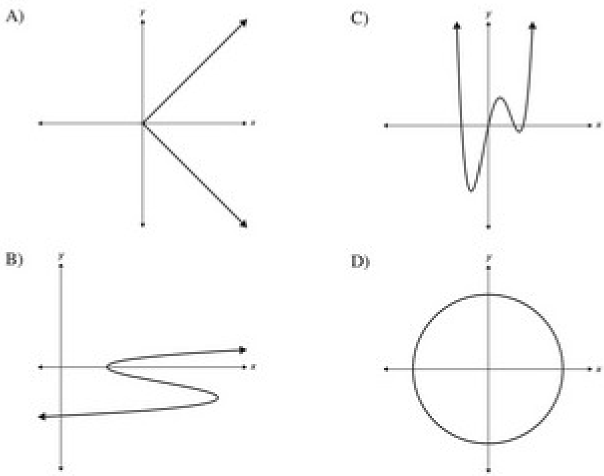
A) $4y$
B) $6y$
C) $10x - 6y$
D) $10x - 4y$

Lesson 34: SAT Practice - General Algebra

SAT Problems of the Week - Heart of Algebra
More on Functions; Other Graphical Representations

Include proof of work that led to your solution.

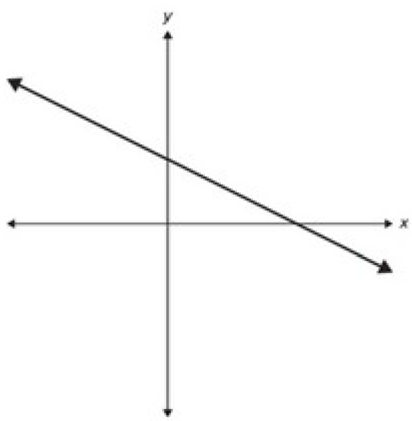
1) Which of the following graphs represents a function?



2) If the graph of the function $f(x) = mx + b$, where m and b are constants, is shifted down a units to create a new function h, what is the equation of h? (Note that a is a constant.)

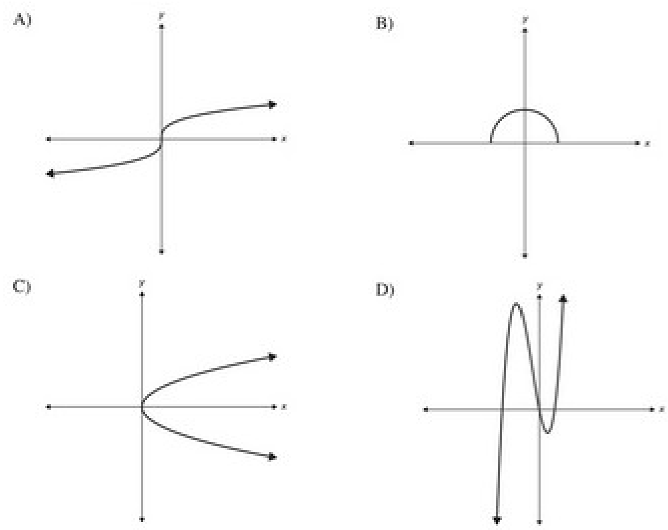
A) $h(x) = mx - b + a$
B) $h(x) = mx + b - a$
C) $h(x) = ax + b$
D) $h(x) = ax + mb$

3) Which of the following best describes the given graph?



A) Positive slope; y-intercept of $(0, 2)$.
B) Negative slope; x-intercept of $(4, 2)$.
C) Negative slope; x-intercept of $(4, 0)$.
D) Positive slope; y-intercept of $(0, 4)$.

4) Which of the given graphs does <u>not</u> represent a function?

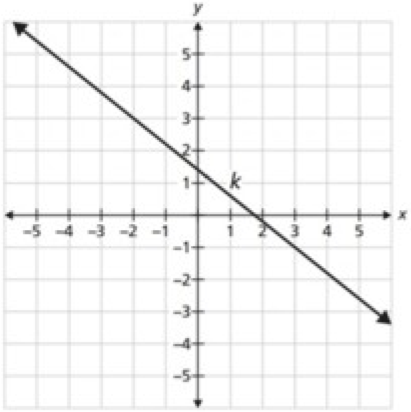


Lesson 34: SAT Practice - General Algebra

SAT Problems of the Week - Heart of Algebra
Slope, Parallel Lines, and Perpendicular Lines

Include proof of work that led to your solution.

1) Find the slope of line k.



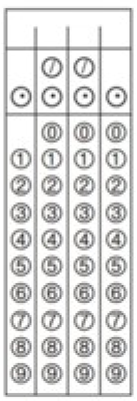
A) $m = \frac{4}{5}$

B) $m = -\frac{4}{5}$

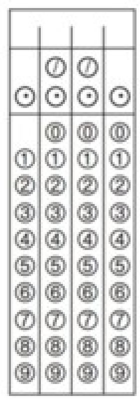
C) $m = \frac{5}{4}$

D) $m = -\frac{5}{4}$

2) Find the slope of the line passing through point $P(-3, 4)$ and point $Q(4, 4)$.



3) Find the value of a so that the slope m of the line through $(-2, 6)$ and $(1, -3a)$ is -2.



4) What is the slope of the line perpendicular to the line $mx + 4y = 17$ if $mx + 4y = 17$ contains the point $\left(\frac{1}{3}, 4\right)$?

5) Find the slope of the line passing through point $C(1, 4)$ and point $D(1, -4)$.

A) $m = 0$

B) $m =$ Undefined

C) $m = 8$

D) $m = -8$

Lesson 34: SAT Practice - General Algebra

SAT Problems of the Week - Heart of Algebra

Creating and Solving Linear Inequalities/Solving Absolute Value Inequalities

Include proof of work that led to your solution.

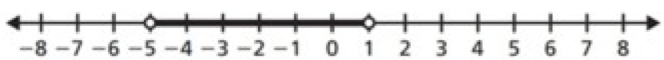
1) Solve the inequality $3x - 8 < 1$.

A) $x > 3$
B) $x > -3$
C) $x < 3$
D) $x < -3$

2) Solve the inequality $1 < 2a + 3 < 7$.

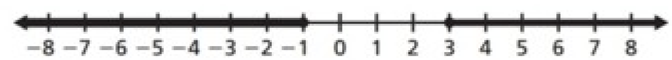
A) $a > -1$ or $a < 2$
B) $-1 > a > 2$
C) $-1 < a < 2$
D) $-1 > a < 2$

3) Match the graph below with the correct inequality.



A) $|2x + 4| < 6$
B) $|2x + 4| > 6$
C) $|2x + 4| < 8$
D) $|2x + 4| > 8$

4) Match the graph below with the correct inequality.



A) $-1 < x < 3$
B) $-1 \le x \le 3$
C) $x < -1$ or $x > 3$
D) $x \le -1$ or $x \ge 3$

5) Which choice is equivalent to $|x| < 5$?

A) $x < 5$
B) $x > -5$
C) $-5 \le x \le 5$
D) $-5 < x < 5$

6) Which of the given choices is equivalent to $|x + 3| > 6$?

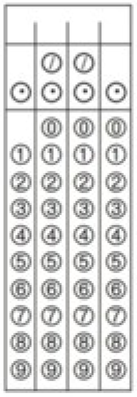
A) $x > 3$ or $x < -3$
B) $x < -3$ and $x > 3$
C) $x < 3$ and $x < -3$
D) $x > 3$ or $x < -9$

Lesson 34: SAT Practice - General Algebra

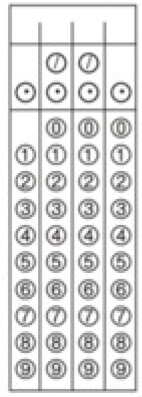
SAT Problems of the Week - Heart of Algebra
Direct and Inverse Variation

Include proof of work that led to your solution.

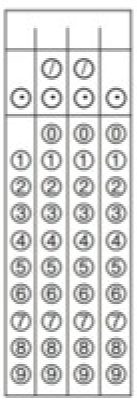
1) If y varies directly as x and $y = 0.5$ when $x = 6$, what is the value of x when $y = 3$?



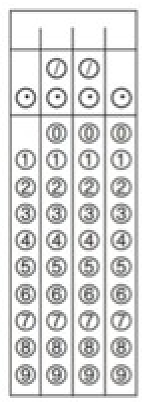
2) If b varies inversely as h and $b = 8$ when $h = 9$, what is the value of b when $h = 6$?



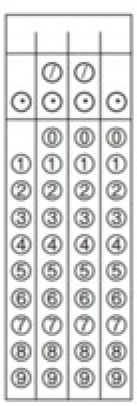
3) If x varies inversely as y and $x = 12$ when $y = 8$, what is the value of x when $y = 10$?



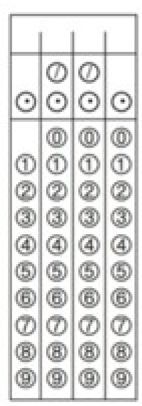
4) If a varies directly as b and $a = 4$ when $b = 9$, what is the value of b when $a = 6$?



5) If x varies directly as the square of y and $x = 12$ when $y = 4$, what is the value of x when $y = 6$?



6) If a varies directly as the square of x and $a = 45$ when $x = 3$, what is the constant of proportionality?



Lesson 34: SAT Practice - General Algebra

SAT Problems of the Week - Heart of Algebra
The xy-Plane, Distance, and Midpoint Formulas

Include proof of work that led to your solution.

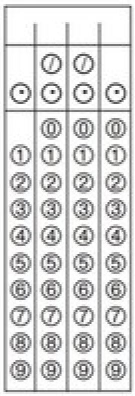
Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

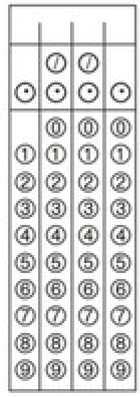
Midpoint Formula:

$$(\bar{x}, \bar{y}) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

1) Find the distance between the points $(1, -1)$ and $(4, -5)$.



2) Find the distance between $(-14, -1)$ and $(-2, -6)$.



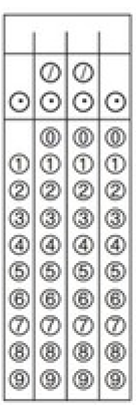
3) Find the midpoint between $(-4, 4)$ and $(4, 0)$.

A) $(0, 0)$
B) $(0, 2)$
C) $(4, 2)$
D) $(8, -4)$

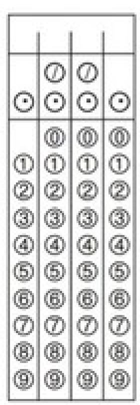
4) Find the endpoint B of $\overline{AB}$ when A is $(1, 3)$ and the midpoint M is $(3, 0)$.

A) $(5, 1.5)$
B) $(4, -2)$
C) $(5, -3)$
D) $(7, -3)$

5) What is the product of the coordinates of the midpoint of the line segment defined by the endpoints *A*(1, 2) and *B*(6, 10)?



6) If one endpoint of a line segment AB is (-5, 2), and the midpoint is (2, -5), what is the x-coordinate of the other endpoint?



Lesson 35: SAT Practice - General Geometry

SAT Problems of the Week

L2-2 Additional Topics in Math – Geometry/Trigonometry/Complex Numbers (No-Calc 3?'s/Calc 3?'s)

⁀ Applying concepts and theorems about lines, angles, triangles, and polygons

Include proof of work that led to your solution.

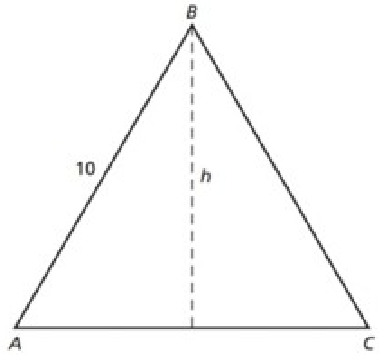
1) The figure below is an equilateral triangle. What is the value of the altitude, h?

A) $5\sqrt{3}$

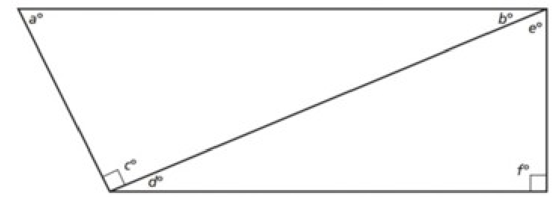
B) $\frac{5\sqrt{3}}{2}$

C) $5\sqrt{2}$

D) $\frac{5\sqrt{2}}{2}$



2) In the figure below, what is the value of $(c + f) - 2(a + b + d + e)$



A) $0^{\sim}$

B) $-90^{\sim}$

C) $-180^{\sim}$

D) $90^{\sim}$

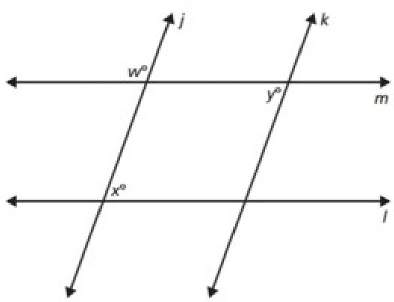
3) In the figure below, j//k and l//m. If $x + y \ll 140^{\sim}$, what is the value of w?

A) 40

B) 110

C) 140

D) 70



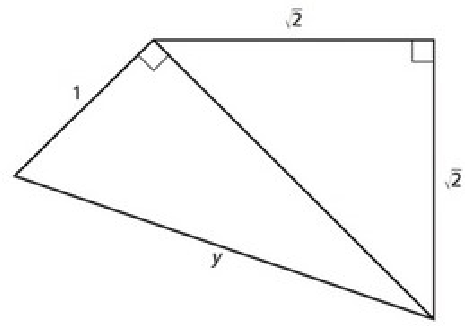
4) In the figure below, what is the value of y?

A) 5

B) $\sqrt{2}$

C) $\sqrt{5}$

D) 1



5) If the sum of the interior angles of a regular polgon equals twice the sum of its exterior angles, how many sides does it have?

A) 3

B) 6

C) 8

D) 12

6) If $x^{\sim}$, $y^{\sim}$, and $z^{\sim}$ are measures of the angles of a triangle, then $\frac{x-1}{2} + \frac{y}{2} + \frac{z+1}{2} \ll$

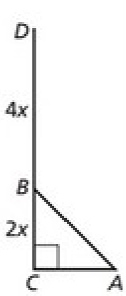
A) $90^{\sim}$

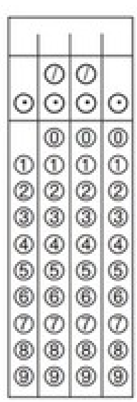
B) $180^{\sim}$

C) $270^{\sim}$

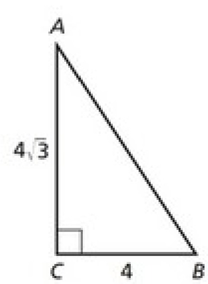
D) $360^{\sim}$

7) In the isosceles right triangle ABC shown below, leg $\overline{AC} \ll 2.25$ units. What is the length of $\overline{CD}$?

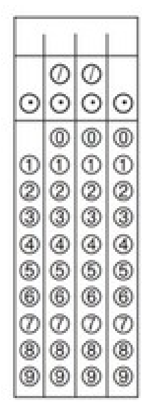




8) In the figure below, what is the measure of AB?



Note: Figure not drawn to scale.



Lesson 35: SAT Practice - General Geometry

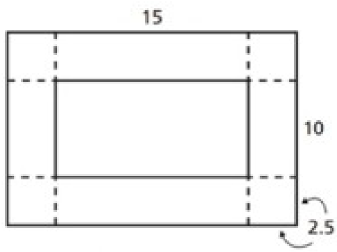
SAT Problems of the Week

L1-1 Additional Topics in Math – Geometry/Trigonometry/Complex Numbers (No-Calc 3?'s/Calc 3?'s)

Using formulas to calculate side length, area, surface area, and volume

Include proof of work that led to your solution.

1) A rectangular piece of tin 10 inches by 15 inches is to be made into a box by cutting out squares that measure 2.5 inches on a side from each corner as shown below. If the sides are then bent up to form the box, what will be the volume of the box in cubic inches?



A) 50 in^3

B) 125 in^3

C) 250 in^3

D) 375 in^3

2) If the ratio of the surface area of sphere A to the surface area of sphere B is $\frac{9}{4}$, what is the ratio of the volume of sphere A to the volume of sphere B? (The formula for the surface area of a sphere is $4\prec\ r^2$.)

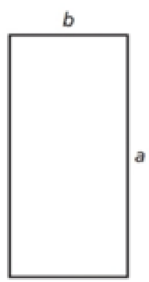
A) $\frac{3}{2}$

B) $\frac{27}{12}$

C) $\frac{9}{8}$

D) $\frac{27}{8}$

3) When the rectangle shown below revolves $360^{\sim}$ about side a, the resulting cylinder has a volume in cubic unites that can be written as



A) $\prec\ ab^2$

B) $\prec\ a^2b$

C) $\prec\ a^2b^2$

D) $\prec\ ab$

4) The rectangular solid base for an office sign is made of concrete and measures 9 feet long, 6 feet wide, and 6 inches high. How much is the cost of the concrete in the pedestal, if concrete costs $90 per cubic yard?

A) $90

B) $270

C) $810

D) $2,340

5) To determine the number of pounds of ice, N, required to reduce the temperature of water in a swimming pool by $F^{\sim}$ Farenheit, use the formula $N \ll 0.31FV$, where V is the volume of the pool (in cubic feet). Ruth has a circular pool with a diameter of 16 feet, which is filled to a depth of 4 feet. Ice is sold in 5-pound bags costing $3.00 each. How much money will Ruth need to reduce the pool's temperature from $84^{\sim}$ to $80^{\sim}$ Farenheit?

A) $30

B) $60

C) $300

D) $600

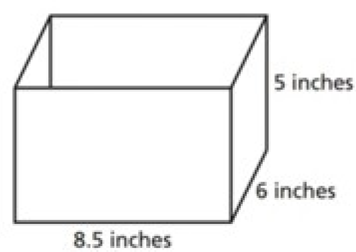
6) A scout troop is packing open cardboard boxes of small toys for a holiday event. Each box has a length of 8.5 inches, a width of 6 inches, and a height of 5 inches. The boxes do not have lids. How much cardboard is needed to make each box?

A) 247 in^2

B) 196 in^2

C) 145 in^2

D) 81 in^2



7) Kiran wants to paint the outside walls of a row of sheds that each measure 10 feet long, 8 feet wide, and 7 feet high. Each gallon of paint covers about 350 ft^2 of surface. How many gallons of paint will Kiran need to paint 5 sheds?

A) 8

B) 7

C) 4

D) 3

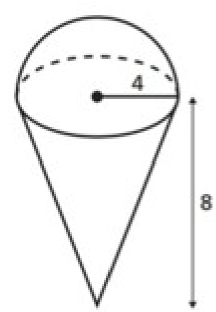
8) Which of the following expressions reflects the volume of the solid shown below?

A) $4\prec \cdot 4^3 \cdot 8$

B) $\frac{2}{3}\prec \cdot 4^3 + \frac{1}{3}\prec \cdot 8 \cdot 4^2$

C) $\frac{\prec \cdot 4^2}{3}(2 \cdot 4 + 8)$

D) $\prec \cdot 4^3 \cdot 8 + \frac{4}{3} \cdot 8 \cdot 4^2$



Lesson 35: SAT Practice - General Geometry

SAT Problems of the Week

L6-1 Additional Topics in Math – Geometry/Trigonometry/Complex Numbers (No-Calc 3?'s/Calc 3?'s)

⌒ Circles in the Coordinate Plane

Include proof of work that led to your solution.

1) Which of the following points does not lie on the circle whose equation is $(x-3)^2+(y-2)^2 \ll 100$?

A) (-7,2)

B) (3,-8)

C) (3,12)

D) (13,1)

2) Which two points are on the circle defined by $(x+2)^2+(y-5)^2 \ll 16$

A) (-2,5) and (2,-5)

B) (-2,1) and (2,5)

C) (-2,5) and (-2,-5)

D) (-2,-5) and (2,5)

3) Write an equation of a circle if the endpoints of the diameter of the circle are (0,3) and (0,-4).

A) $x^2+(y-\frac{7}{2})^2 \ll \frac{1}{4}$

B) $x^2+(y+\frac{1}{2})^2 \ll \frac{49}{4}$

C) $x^2+(y+\frac{1}{4})^2 \ll \frac{7}{2}$

D) $x^2+(y-\frac{1}{2})^2 \ll \frac{1}{4}$

4) Find the circumference of a circle whose equation is $2x^2+2y^2 \ll 72$

A) $36\prec$

B) $12\prec$

C) $72\prec$

D) $144\prec$

5) A circle centered at te origin passes through the point (2,4). Write the equation of the circle.

A) $x^2+y^2 \ll 20$

B) $x^2+y^2 \ll 400$

C) $(x-2)^2+(y-4)^2 \ll 20$

D) $(x-2)^2+(y-4)^2 \ll 400$

6) The equation of a circle is $(x-8)^2+(y+11)^2 \ll 25$. Find the length of radius and the coordinates of the center.

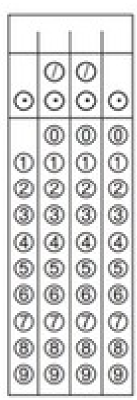
A) radius 25, center (8,-11)

B) radius 25, center (-8,11)

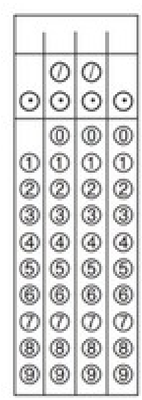
C) radius 5, center (-8,11)

D) radius 5, center (8,-11)

7) A wheel whose radius measures 18 inches is rotated. If a point on the circumference of the wheel moves through an arc of 10 inches, what is themeasure, in radians, of the angle through which a spoke of a wheel travels?



8) In standard position, an angle of $-\frac{\prec}{3}$ radians has the same terminal angle as what degree measure?



Lesson 35: SAT Practice - General Geometry

SAT Problems of the Week

L3-2 Additional Topics in Math – Geometry/Trigonometry/Complex Numbers (No-Calc 3?'s/Calc 3?'s)

Using circle theorems to find arc length, angle measures, chord lengths, and sector areas

Include proof of work that led to your solution.

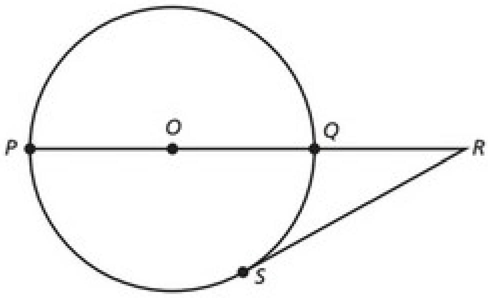
1) In the figure below, $\overline{RS}$ is tangent to circle O. If $\overline{PO}$ is 5 and $\overline{QR}$ is 8, what is the value of $\overline{RS}$?

A) 5

B) 12

C) $\sqrt{39}$

D) $\sqrt{8}$



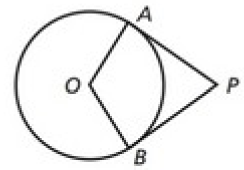
2) Given: $\overline{PA}$ and $\overline{PB}$ are tangent to circle O at points A and B, respectively. Which of the following is *always* true?

A) $m\angle P \ll m\angle AOB$

B) $m\angle P \succ m\angle AOB$

C) $m\angle P + m\angle AOB \ll 90^{\sim}$

D) $m\angle P + m\angle AOB \ll 180^{\sim}$



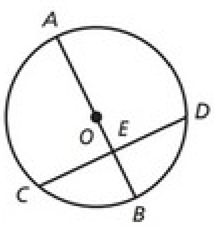
3) In circle O, chord $\overline{CD}$ is perpendicular to diameter $\overline{AOB}$ at point E. If $\overline{OD} \ll 17$ and $\overline{CD} \ll 30$, what is the length of $\overline{EB}$?

A) 8

B) 9

C) 10

D) 15



4) Two chords intersect inside a circle. The lengths of the segments of the first chord are 6 and 15, and the full length of the second chord is 23. The lengths of the segments of the second chord could be:

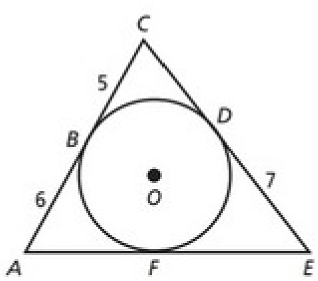
A) 5 and 18

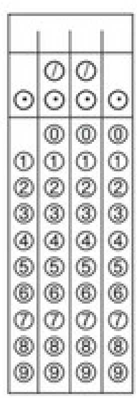
B) 3 and 20

C) 11.5 and 11.5

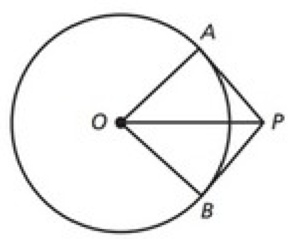
D) 10 and 13

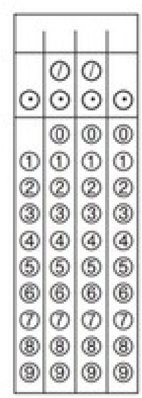
5) Circle O is inscribed in λACE as shown below. If $AB \ll 6$ inches, $BC \ll 5$, and $DE \ll 7$ inches, what is the perimeter of λACE , in inches?





6) $\overline{PA}$ and $\overline{PB}$ are tangents drawn to circle O from point P. The measure of $\angle AOP$ is $40^{\sim}$. What is $m\angle APB$?





Lesson 36: Origins of Pi

Where does PI Come From?

Resources Needed:

- String
- Ruler (metric units preferred)
- Calculator (your phone calculator will work)
- 5 different sized circular objects
- 1 oreo cookie (or choose your own favorite circular shaped food)
- Compass (optional)

Directions: Choose five different sized circular objects and your cookie/circular food item (plates, soup cans, bike tires, etc).. You can also use a compass to carefully draw at least 5 different sized circles if you have one at home. Fill in the table by measuring the diameter and circumference. To measure the circumference, use the string to approximate the circumference of the circle (you may need help with someone at home with this step. Please measure in cm/mm. Once you have measured your 5 circles, do the same for the cookie/circular food item. Once you finish measuring go ahead and eat your cookie while answering the questions below.

	Diameter (distance across the circle through the center)	Circumference (distance around the circle)	$\frac{Circumference}{Diamater}$ (Use your calculator)
Object # 1			
Object # 2			
Object # 3			
Object # 4			
Object # 5			
Cookie/other food item			

1. What do you notice about the last column?

2. Press the π button on your calculator. What do you notice? Is this number similar to any of the numbers in your data table above? Can you think of any reasons why there is not an exact match?

3. The circumference of the Earth is 24,901 miles and the diameter is 7917.5 miles. Divide the Earth's circumference by its diameter. What do you notice?

4. What do you predict would happen if you could measure the circumference and diameter of any circle in the world? Does the size of the object matter? Explain your reasoning.

Lesson 37: Number Sense Games

Part 1

2s Challenge - Using exactly four twos, add arithmetical symbols between the twos to make each of the target numbers. You may use plus, minus, times, and divide symbols, as well as parentheses and brackets for grouping.

0 = (2-2)/(2+2)

1 =

2 =

3 =

4 =

5 =

6 =

10 =

12

3s Challenge - Using exactly four threes, add arithmetical symbols between the twos to make each of the target numbers. You may use plus, minus, times, and divide symbols, as well as parentheses and brackets for grouping.

3 =

4 =

5 =

6 =

7 =

8 =

9 =

10 =

5s Challenge - Using exactly four fives, add arithmetical symbols between the twos to make each of the target numbers. You may use plus, minus, times, and divide symbols, as well as parentheses and brackets for grouping.

3 =

5 =

6 =

26 =

30 =

50 =

55 =

120 =

9s Challenge - Using exactly four threes, add arithmetical symbols between the twos to make each of the target numbers. You may use plus, minus, times, and divide symbols, as well as parentheses and brackets for grouping.

7 =

9 =

10 =

19 =

80 =

81 =

90 =

720 =

Lesson 37: Number Sense Games

Part 2

Sudoku

The classic Sudoku game involves a grid of 81 squares. The grid is divided into nine blocks, each containing nine squares.

The rules of the game are simple: each of the nine blocks has to contain all the numbers 1-9 within its squares. Each number can only appear once in a row, column or box.

The difficulty lies in that each vertical nine-square column, or horizontal nine-square line across, within the larger square, must also contain the numbers 1-9, without repetition or omission.

Every puzzle has just one correct solution.

Level 1

						1		
4			9	1		7	8	
			7	6		2		3
9				4	5		1	
	7			2			3	
	1		6	9				2
2		7		8	3			
	4	8		5	6			9
		6						

Level 2

5			8		1	4	6	
				2	5	3		1
8							7	
	9			4	3			
3								5
			7	9			3	
	4							8
1		2	4	5				
	3	5	1		7			4

Level 3

8	9		2		1		3	
		4	3					
	2	3						6
9		5	8				6	4
3	4				6	9		5
4						6	2	
					8	5		
	3		6		5		7	9

Level 4

		6					8	
		4	9			5		
	3				5	6		7
		9	1	8		3		5
3				5				2
6		2		3	9	8		
4		3	2				5	
		5			6	2		
	6					1		

Lesson 37: Number Sense Games

Part 3

What two numbers…? Determine what two integers multiply and add to the given numbers. For example: What two numbers multiply to 1 and add to 2? The answer would be 1 and 1.

What two numbers multiply to 5 and add to 6?

What two numbers multiply to 10 and add to 7?

What two numbers multiply to 42 and add to 23?

What two numbers multiply to 48 and add to 14?

What two numbers multiply to 70 and add to 37?

What two numbers multiply to 99 and add to 36?

What two numbers multiply to -3 and add to 2?

What two numbers multiply to -12 and add to 4?

What two numbers multiply to -21 and add to 4?

What two numbers multiply to -49 and add to 0?

What two numbers multiply to -60 and add to 17?

What two numbers multiply to -120 and add to 10?

What two numbers multiply to -6 and add to -5?

What two numbers multiply to -15 and add to -2?

What two numbers multiply to -36 and add to 0?

What two numbers multiply to -55 and add to -6?

What two numbers multiply to - 72and add to -1?

What two numbers multiply to -120 and add to -40?

What two numbers multiply to 7 and add to -8?

What two numbers multiply to 18 and add to -11?

What two numbers multiply to 28 and add to -11?

What two numbers multiply to 63 and add to -24?

What two numbers multiply to 84 and add to -25?

What two numbers multiply to 144 and add to -30 ?