Middle School Math Enrichment

Wayne-Westland
COMMUNITY SCHOOLS

Hopefully everyone is staying healthy and safe during this time. Please use these activities to enrich your understanding of Mathematics during this time.

The lessons can be done in any order. They are just titled so you can easily find them

Lesson 1: Subtracting Whole Numbers

1. 1,100 − 485	2. 200 − 135	3. 1,800 − 957	4. 900 − 224	5. 800 − 294
6. 300 − 214	7. 500 − 385	8. 1,000 − 739	9. 700 − 434	10. 900 − 726
11. 300 − 294	12. 1,700 − 965	13. 400 − 351	14. 700 − 635	15. 1,000 − 835
16. 1,910 − 69	17. 1,403 − 44	18. 1,339 − 14	19. 1,874 − 34	20. 1,100 − 13
21. 790 − 70	22. 877 − 92	23. 469 − 49	24. 1,552 − 20	25. 1,856 − 47
26. 1,618 − 65	27. 1,642 − 17	28. 759 − 77	29. 42 − 17	30. 1,420 − 37

Lesson 2: Multiplying Whole Numbers

1. 576 × 96	2. 886 × 48	3. 780 × 54	4. 919 × 50	5. 350 × 81
6. 473 × 42	7. 445 × 93	8. 161 × 89	9. 679 × 39	10. 122 × 22
11. 288 × 23	12. 647 × 59	13. 582 × 35	14. 917 × 51	15. 562 × 57
16. 18 × 50	17. 568 × 50	18. 942 × 30	19. 186 × 50	20. 933 × 50
21. 352 × 90	22. 118 × 70	23. 632 × 90	24. 823 × 30	25. 308 × 20
26. 293 × 80	27. 418 × 60	28. 320 × 20	29. 23 × 80	30. 621 × 20

Lesson 3: Dividing Whole Numbers

1. $70\overline{)630}$

2. $99\overline{)594}$

3. $36\overline{)720}$

4. $49\overline{)147}$

5. $29\overline{)667}$

6. $19\overline{)437}$

7. $34\overline{)578}$

8. $34\overline{)646}$

9. $16\overline{)112}$

10. $39\overline{)312}$

11. $50\overline{)341}$

12. $40\overline{)350}$

13. $90\overline{)277}$

14. $19\overline{)350}$

15. $69\overline{)661}$

16. $51\overline{)466}$

17. $17\overline{)210}$

18. $45\overline{)800}$

19. $37\overline{)227}$

20. $35\overline{)592}$

Use Multiplication and Division Equations to Solve Problems

Multiplication and division are inverse operations. Inverse operations undo each other. Use inverse operations to solve an equation.

> **Division Property of Equality**
>
> To solve a multiplication equation, divide both sides of the equation by the same number.
>
> **Multiplication Property of Equality**
>
> To solve a division equation, multiply both sides of the equation by the same number.

Mr. Folsom buys 8 boxes of ice pops. There are the same number of ice pops in each box. There are 96 ice pops in all. How many ice pops are in each box?

A. Write what you know.
There are 8 boxes of ice pops.
There are 96 ice pops in all.

B. Write an equation to represent the problem.
Let p represent the number of ice pops in each box.
$8p = 96$

C. Isolate the variable by using the Division Property of Equality.
Divide both sides of the equation by 8.
$\frac{8p}{8} = \frac{96}{8}$
$p = 12$
There are 12 ice pops in each box.

Solve the equation.

1. $5x = 80$
$x =$ ____

2. $\frac{y}{6} = 9$
$y =$ ____

3. $\frac{2}{3}z = 18$
$z =$ ____

4. Explain how to solve $3b = 48$. Give the solution in your explanation.

____

5. Explain how to solve $\frac{c}{9} = 3$. Give the solution in your explanation.

____

6. Yasmine earns $54 for dog-walking. She walks her neighbor's dog 12 times.

A. Write an equation to represent how to find the amount Yasmine earns per dog walk. Explain what your variable represents.

____

B. How much does Yasmine earn per dog walk?

____

Lesson 5: Write and Solve Equations

Match and paste the variable description and equation for each problem. Then solve the equation, showing your work.

Name ___________________

Cut along the dotted line and cut out each parallelogram

Problem	Define the Variable / Equation	Equation and Work / Solution in a Sentence
Christa is saving for a new tablet. It costs $70, including tax. She has already saved $35. How much more does she need to save?	Define the Variable Equation	Equation and Work Solution in a Sentence
Bella bought packs of stickers to give to her friends. Each pack cost $4. She spent a total of $12 on stickers. How many packs did she buy?	Define the Variable Equation	Equation and Work Solution in a Sentence
Robert had money in his savings account. He spent $35 of it on a new jacket. He has $70 left. How much money did he have in his savings before he bought the jacket?	Define the Variable Equation	Equation and Work Solution in a Sentence
Pete played in two basketball games. He scored 4 fewer points in the second game than he scored in the first. He scored 12 points in the second game. How many points did he score in the first game?	Define the Variable Equation	Equation and Work Solution in a Sentence
Bella brought stickers to give to 12 of her friends. Each friend received 4 stickers. How many stickers did she bring to school?	Define the Variable Equation	Equation and Work Solution in a Sentence

- *Let x represent the amount of money in the savings account* — $\frac{x}{4} = 12$
- *Let x represent the amount of money that needs to be saved* — $4x = 12$
- *Let x represent the number of stickers brought to school* — $x + 35 = 70$
- *Let x represent the number of points scored in the first game* — $x - 4 = 12$
- *Let x represent the number of packs of stickers bought* — $x - 35 = 70$

Apply Two-Step Equations to Solve Problems

The **solution of an equation** is the value of the variable that makes the equation true. You can use the properties of operations and the properties of equality to solve a two-step equation to find its solution.

Brody is 4 years older than his brother Logan. Together, they are 14 years old. How old is Logan?

A. Write an equation to represent the situation.

$x + (x + 4) = 14$
$2x + 4 = 14$

B. Use the Subtraction Property of Equality to isolate the variable term.

$$\begin{aligned} 2x + 4 &= 14 \\ -4 &\quad -4 \\ \hline 2x &= 10 \end{aligned}$$

C. Use the Division Property of Equality to solve for x.

$$\frac{2x}{2} = \frac{10}{2}$$
$$x = 5$$

D. State your answer.

Logan is 5 years old.

1. The formula $C = \frac{5}{9}(F - 32)$ can be used to convert temperature from Celsius (C) to Fahrenheit (F). The highest temperature ever recorded in the United States was 134 °F in Death Valley on July 10, 1913. What was the temperature that day in Celsius, rounded to the nearest degree?

____

2. Mia made three times as many baskets in yesterday's basketball game as she did last Saturday. She scored a total of 24 baskets for both games. How many baskets did Mia score last Saturday?

____

Solve each equation. Check your solution.

3. $\frac{a}{4} + 2 = 6$

____

4. $9x - 7 = 20$

____

5. $-9x + 1 = -80$

____

6. $2.6x + 10.5 = 31.3$

____

Two-Step Equation Maze!

Directions: Use your solutions to navigate through the puzzle. **SHOW ALL STEPS!!!!**

Start! $4x + 10 = -26$

-4

$\frac{x}{3} + 10 = 15$

15

$9 - 2x = 35$

-13

$\frac{2}{3}x + 15 = 17$

9 -9 -8 -2 48 -22 3

$\frac{x-7}{4} = -2$

14

$\frac{1}{2}x + 13 = 9$

8

$\frac{3}{4}x - 9 = 27$

4

$-5x - 10 = 10$

-14 24 44 11 -4 -4 0

$8 - \frac{1}{3}x = 16$

6

$-12x - 17 = -89$

5

$-8 = \frac{x+11}{-2}$

-5

$19 - \frac{5}{2}x = 34$

-72 -24 -6 17 4 48 -6

$28 - 32x = 92$

7

$5 - x = 12$

-7

$13 - \frac{3}{2}x = 37$

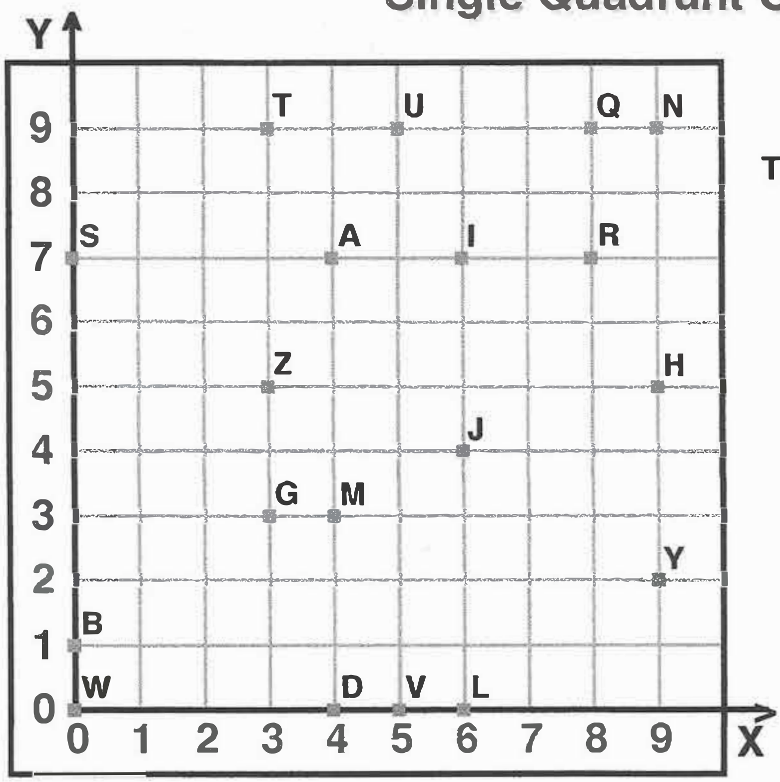
-16

END! ☺

______________________________ ____________

______________________________ ____________

Single Quadrant Ordered Pairs



Tell what point is located at each ordered pair.

1) (4,3) _____

2) (3,9) _____

3) (5,9) _____

4) (9,2) _____

5) (6,4) _____

6) (0,0) _____

7) (8,7) _____

8) (3,3) _____

9) (8,9) _____

10) (9,5) _____

Write the ordered pair for each given point.

11) **D** _____

12) **A** _____

13) **V** _____

14) **S** _____

15) **N** _____

16) **L** _____

17) **Z** _____

18) **B** _____

19) **I** _____

Plot the following points on the coordinate grid.

20) **X** (1,2)

21) **O** (4,9)

22) **P** (4,8)

23) **F** (9,3)

24) **E** (5,6)

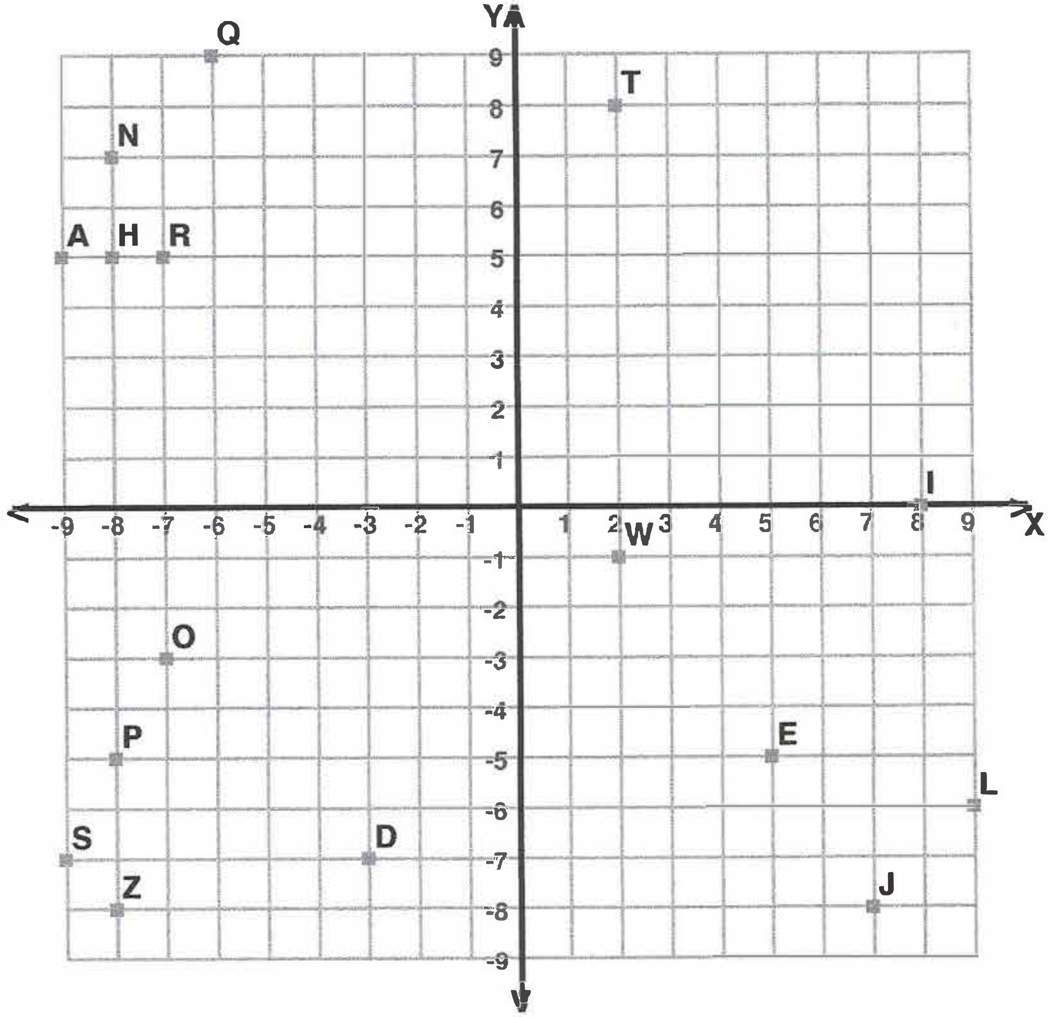
25) **K** (8,4)

Lesson 9: Ordered Pairs in 4 Quadrants

______________________________ ____________

______________________________ ____________

Four Quadrant Ordered Pairs



Tell what point is located at each ordered pair.

1) (-9,-7) _____	3) (+5,-5) _____	5) (-7,+5) _____	7) (-9,+5) _____
2) (-7,-3) _____	4) (-8,+7) _____	6) (-8,-5) _____	8) (+9,-6) _____

Write the ordered pair for each given point.

9) **J** _______	11) **Z** _______	13) **Q** _______	15) **W** _______
10) **I** _______	12) **D** _______	14) **T** _______	16) **H** _______

Plot the following points on the coordinate grid.

17) **M** (-1,+9)	19) **C** (-2,+6)	21) **K** (+6,+3)	23) **B** (-8,-1)
18) **F** (-9,-3)	20) **U** (+4,-3)	22) **V** (+7,-6)	24) **X** (-9,+6)

Lesson 10: Graphing Practice

Coordinate Graphing Mystery Picture - First Quadrant

Plot the ordered pairs and connect them with a straight line as you plot.

START
(24,22)
(26,22)
(26,24)
(24,24)
(24,22)
STOP

START
(16,20)
(18,20)
(17,24)
(16,20)
STOP

START
(4,28)
(7,32)
(12,31)
(8,34)
(10,38)
(7,36)
(4,40)
(5,35)
(1,34)
(5,33)
(4,28)
STOP

START
(25,11)
(26,14)
(29,11)
(28,15)
(31,14)
(29,18)
(33,19)
(29,22)
(33,24)
(29,25)
(30,29)
(27,27)
(26,30)
(24,27)
(23,28)
STOP

START
(14,11)
(15,14)
(17,10)
(18,14)
(20,10)
(21,12)
STOP

START
(22,39)
(18,33)
(20,29)
(17,28)
(18,26)
(15,26)
(13,15)
(16,15)
(16,18)
(18,18)
(18,15)
(21,15)
(19,27)
(23,28)
(20,33)
(22,39)
STOP

START
(12,12)
(10,14)
(8,11)
(7,14)
(4,10)
(5,14)
(1,13)
(4,16)
(1,18)
(5,19)
(1,22)
(5,23)
(2,26)
(6,27)
(7,30)
(9,27)
(12,30)
(14,26)
(16,30)
(17,28)
STOP

START
(14,1)
(17,3)
(20,1)
(18,4)
(20,6)
(18,6)
(17,9)
(16,6)
(14,6)
(16,4)
(14,1)
STOP

START
(24,16)
(24,20)
(27,20)
(28,21)
(28,25)
(27,26)
(22,26)
(22,15)
(24,15)
(21,12)
(24,10)
(21,8)
(26,1)
(24,8)
(27,10)
(23,12)
(27,17)
(24,16)
STOP

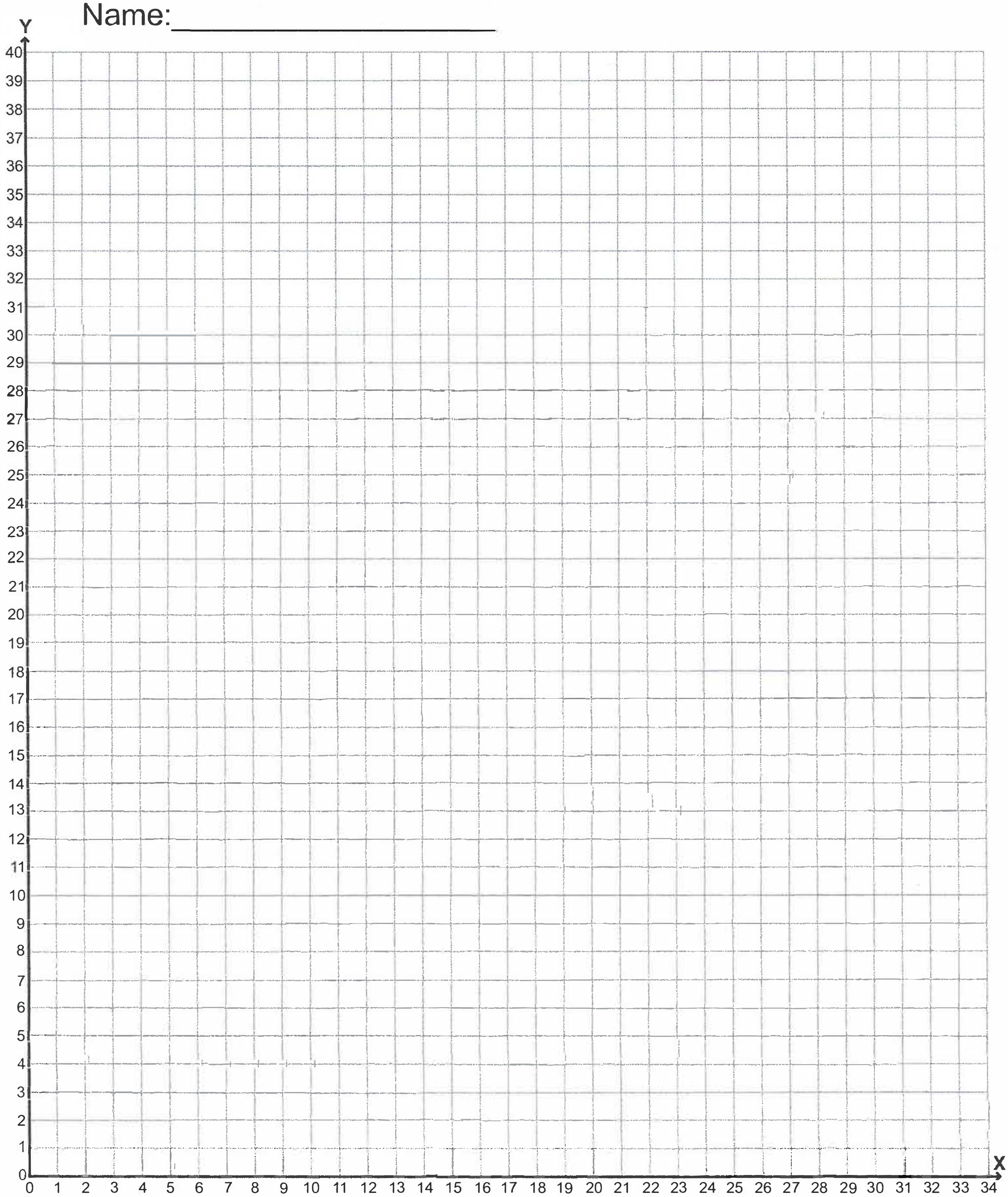
START
(26,37)
(29,36)
(32,39)
(31,35)
(34,34)
(31,33)
(32,28)
(29,32)
(24,31)
(28,34)
(26,37)
STOP

START
(7,1)
(13,6)
(10,10)
(14,11)
(11,17)
(8,17)
(13,26)
(5,26)
(6,24)
(9,24)
(4,15)
(11,15)
(12,12)
(8,10)
(11,6)
(7,1)
STOP

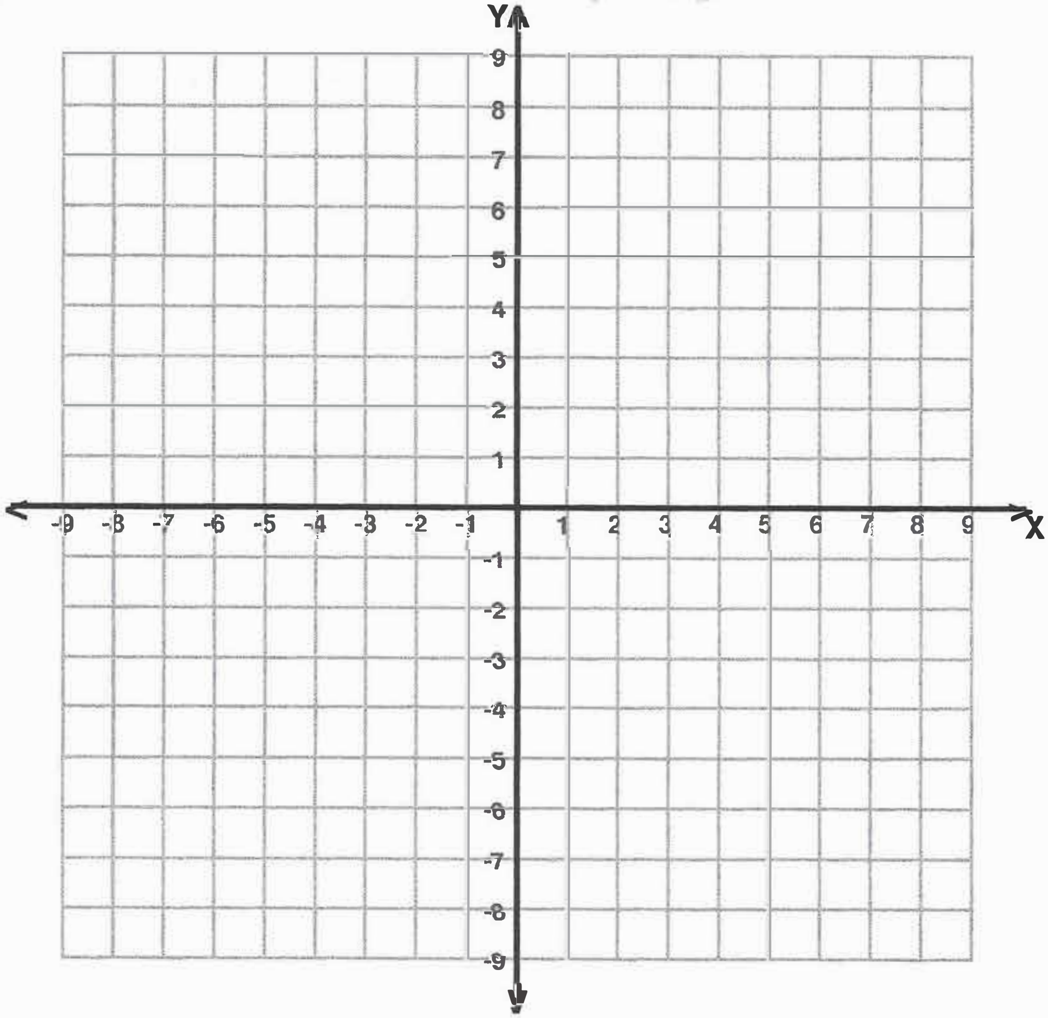
Lesson 10: Graphing Practice

Coordinate Graphing Mystery Picture - First Quadrant

Name:____________________



Four Quadrant Graphing Puzzle



Connect each sequence of points with a line.

(-3,0) , (-4,-1) , (-6,-2) , (-6,-3) , (-5,-4) , (-4,-4) , (-3,-4) , (1,-2) End of Sequence

(-1,1) , (5,3) , (6,5) , (7,5) , (7,3) , (9,2) , (8,1.5) , (6,2) , (2,-.5) End of Sequence

(-5,-3) , (-5.5,0) , (-5,0) , (-5,-6) , (-4.5,-6) , (-5,-3) End of Sequence

(-9,3) , (-7,4) , (7,-3) , (5,-4) , (-9,3) End of Sequence

(-6,-2) , (-4,-3) , (-2,-2) , (-1,-1) End of Sequence

(5,3) , (4,4) , (4.5,4.5) , (5.5,4) End of Sequence

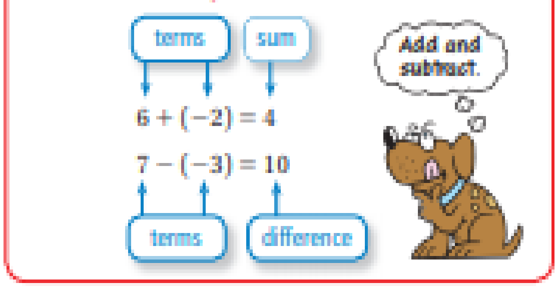
(-4,-1) , (-2,-2) End of Sequence

What is the shape ? ____________

REVIEW: Adding and Subtracting Integers

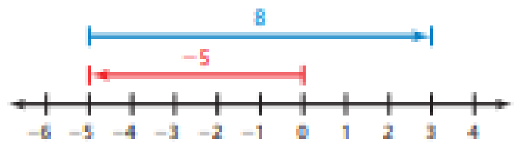
Name ______________________________

Key Concept and Vocabulary



Visual Model

Positive numbers involve movement to the right. Negative numbers involve movement to the left.



$-5 + 8 = 3$

Skill Examples

1. $5 + (-3) = 2$
2. $5 - (-2) = 5 + 2 = 7$
3. $-2 + 4 = 2$
4. $-3 - (-2) = -3 + 2 = -1$
5. $8 - (-3) = 8 + 3 = 11$

To subtract, add the opposite.

Application Example

6. The temperature is 8°F in the morning and drops to −5°F in the evening. What is the difference between these temperatures?

$$8 - (-5) = 8 + 5$$
$$= 13$$

The difference is 13 degrees.

PRACTICE MAKES PURR-FECT®

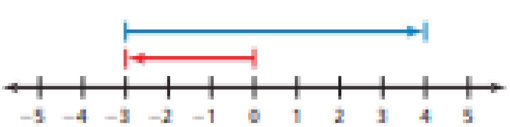
Check your answers at BigIdeasMath.com.

Find the sum or difference.

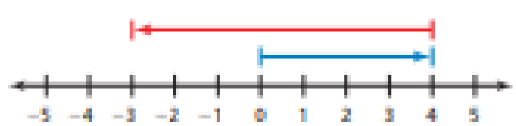
7. $-2 + 3 =$ _____
8. $-4 - 5 =$ _____
9. $8 - 2 =$ _____
10. $8 - (-2) =$ _____
11. $-4 - (-1) =$ _____
12. $-5 + (-5) =$ _____
13. $4 - (-8) =$ _____
14. $4 - 8 =$ _____
15. $-4 + (-6) =$ _____
16. $-4 - (-6) =$ _____
17. $10 - 13 =$ _____
18. $13 - (-10) =$ _____

Write the addition or subtraction shown by the number line.

19.

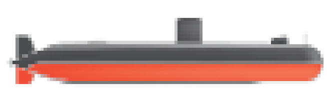


20.



21. **TEMPERATURE** The temperature is 16°F in the morning and drops to −15°F in the evening. What is the difference between these temperatures? ____________

22. **SUBMARINE** A submarine is 450 feet below sea level. It descends 300 feet. What is its new position? Show your work.



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Name ______________________________ Date __________

Lesson 1.2 Reteach (continued)

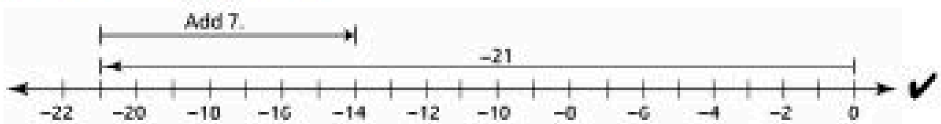
b. Find −21 + 7.

$-21 + 7 = -14$ $\quad |-21| > |7|$. So, subtract $|7|$ from $|-21|$.

Use the sign of −21.

▶ The sum is −14.

CHECK: Use a number line.



You can find the sum of three integers by using what you have learned to add the first two integers, and then add the result to the third integer. You can also use the Commutative and Associative Properties of Addition to find sums of integers.

Find the sum. Use integer counters or a number line to verify your answer.

1. $23 + 41$ **2.** $17 + (-2)$ **3.** $(-3) + 12$ **4.** $(-5) + (-6)$

5. $(-13) + (-13)$ **6.** $(-8) + 10$ **7.** $4 + (-9)$ **8.** $(-7) + 7$

9. $7 + (-1)$ **10.** $(-30) + 4$ **11.** $(-5) + (-6)$ **12.** $(-2) + (-3)$

Find the sum.

13. $(-10) + (-5) + 1$ **14.** $(-12) + 8 + (-11)$ **15.** $19 + (-7) + 3$

16. $(-1) + 1 + (-2)$ **17.** $(-8) + (-3) + 11$ **18.** $14 + 5 + (-16)$

Name ______________________________ Date ______

Lesson 1.2 **Enrichment and Extension**

Magic Squares with Integers

According to a legend, the Chinese Emperor Yu-Huang saw a magic square on the back of a turtle. In a *magic square*, the sum of the numbers in each row, column, and diagonal are the same. This sum is called the magic sum.

This magic square uses integers −6 to 2 exactly once. The magic sum is −6.

1	−6	−1
−4	−2	0
−3	2	−5

Diagonal 1: $-3 + (-2) + (-1) = -6$

Row 1: $1 + (-6) + (-1) = -6$

Row 2: $-4 + (-2) + 0 = -6$

Row 3: $-3 + 2 + (-5) = -6$

Diagonal 2: $1 + (-2) + (-5) = -6$

Column 1: $1 + (-4) + (-3) = -6$

Column 2: $-6 + (-2) + 2 = -6$

Column 3: $-1 + 0 + (-5) = -6$

Complete the magic square using each integer only once. The magic sum is given.

1. Use −9 to −1; Magic Sum = −15

−8		−4
	−7	

2. Use −5 to 3; Magic Sum = −3

−2		
		1
	−5	

3. Use −7 to 8; Magic Sum = 2

	7		−4
		−1	
0		3	−3
	−5		

4. Use −10 to 5; Magic Sum = −10

−4	1		
−9		−3	0
	−8		−6
		4	

5. Create your own magic square with integers having the magic sum 6.

Lesson 14: Multiplication of Integers

No Calculator

Name ______________________________

PRACTICE

Simplify.

1. $2 \times {}^{-}4$ ______
2. ${}^{-}4 \times 4$ ______
3. ${}^{-}7 \times {}^{-}3$ ______
4. ${}^{-}2 \times 0$ ______
5. ${}^{-}3 \times 9$ ______
6. 2×8 ______
7. ${}^{-}6 \times {}^{-}3$ ______
8. $4 \times {}^{-}3$ ______
9. ${}^{-}1 \times {}^{-}8$ ______
10. ${}^{-}5 \times 6$ ______
11. $3 \times {}^{-}7$ ______
12. $6 \times {}^{-}6$ ______
13. ${}^{-}9 \times {}^{-}7$ ______
14. ${}^{-}7 \times {}^{-}8$ ______
15. ${}^{-}12 \times {}^{-}5$ ______
16. ${}^{-}4 \times {}^{-}5$ ______
17. ${}^{-}2 \times 4 \times 2$ ______
18. ${}^{-}5 \times {}^{-}6 \times 1$ ______
19. ${}^{-}2 \times {}^{-}2 \times {}^{-}2$ ______
20. ${}^{-}2 \times 4 \times {}^{-}6$ ______
21. ${}^{-}2 \times |{}^{-}5|$ ______
22. $|{}^{-}4| \times {}^{-}5$ ______
23. $|{}^{-}2| \times |{}^{-}6|$ ______
24. $|{}^{-}3| \times {}^{-}|{}^{-}3|$ ______

Estimate. Be sure the sign is correct.

25. ${}^{-}201 \times 3$ ________
26. ${}^{-}249 \times {}^{-}2$ ________
27. $5 \times {}^{-}198$ ________
28. ${}^{-}503 \times {}^{-}4$ ________
29. ${}^{-}59 \times {}^{-}203$ ________
30. ${}^{-}397 \times 204$ ________
31. ${}^{-}29 \times {}^{-}301$ ________
32. $11 \times {}^{-}196$ ________
33. ${}^{-}19 \times {}^{-}498$ ________

Solve.

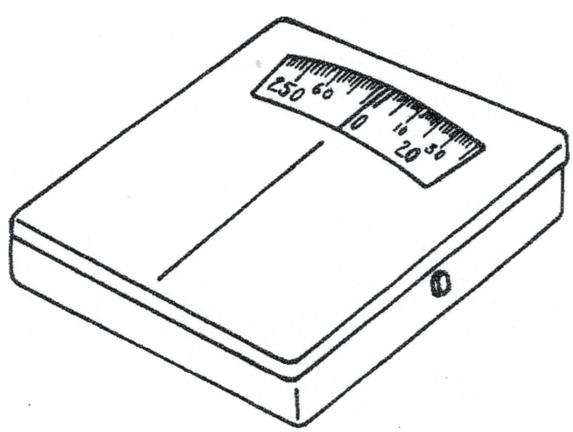
34. A shark was swimming at a depth of 2 m. Startled by a boat, the shark increased its depth by 5 times. What was its new depth?

    ____

    If you represented the surface as 0, how would you represent the shark's initial depth? ______

35. While on a diet, Bessie's weight loss averaged 8 lb per month for 4 months. What was the absolute value of the number of pounds she lost?

    ____



No Calculator

Why isn't your nose twelve inches long?

DIRECTIONS: Solve each equation below. Then find your answer in the decoder. Each time your answer occurs in the decoder, write the letter of the problem above it.

1. (-9)(18) = (n)
2. (-8)(-8)(-3) = (i)
3. (5)(-7)(-4) = (l)
4. (10)(-17)(5) = (h)
5. -11(-7) = (a)
6. -8(-8)(-8) = (o)
7. (-6)(-4)(3) = (u)
8. -16(-2) = (d)
9. -7(30) = (w)
10. -6(-7)(2) = (b)
11. (21)(-3)(2) = (e)
12. 6(-3)(-5) = (t)
13. -3(-4)(-9) = (f)

90 -850 -126 -162 -192 90 -210 -512 72 140 32

84 -126 77 -108 -512 -512 90

Lesson 16: Distributive Property 1

Lesson 3 Skill: The Distributive Property **Practice:** Multiply both terms inside the parentheses by the number outside of the parentheses. (Work left to right.)

Use the Distributive Property to rewrite each expression.

1.) 3(x + 2)
3x + 6
3 · x   Keep   3 · 2

2.) 4(y + 5)

3.) 6(b + 2)

4.) 4(7 + c)

5.) 8(2 + t)

6.) 7(3 +w)

7.) 15(a + b)
15a + 15b
15 · a   Keep   15 · b

8.) 20(c + d)

9.) 72(w + x)

10.) (x + 10)9

11.) (y + 5)10   Go left to right!

12.) (6 + f)3

13.) 4(g + 12)

14.) 12(5 + y)

15.) (m + 4)6

16.) 12(y + z)

17.) (m + n)3

18.) (9)(h +4)

19.) 4(a – 10)

20.) 5(m – 8)

21.) 10(8 –z)

22.) 2(2a + 6)

23.) 5(3h + 6)

24.) 4(6r + 7s)

25.) 7(3a – 6)

26.) 6(4b – 10)

27.) (7a – 3)(8)

28.) 3(5z +10)

29.) 6(5b – 7)

30.) (3 + z)10

31.) 2(a – b)

32.) 3(3h +12)

33.) 4(5 +10r)

34.) (g – 3)12

35.) 7(w + z)

36.) 6(9 +c)

37.) 4(y – 7)

38.) 5(9 –w)

39.) (6 + r)(8)

Lesson 17: Distributive Property 2

Name ______________________________ Hour _____ Date ______________

Module 3 Lesson 3 Practice: Writing Products as Sums and Sums as Products

Rewrite each expression in standard form. Draw the arrows to show how you used the distributive property and show the steps!

1. 4(2x + 6y)
   4(2x) + 4(6y)
   8x + 24y
2. 2(3x – 3y)
3. -6(2x + 3y)
4. 7(- x + 6y)
5. 2(x – 12)
6. 12(2x + 5y)
7. 3(2 + x)
8. 3(x – 4)
9. 8(y – 2x)
10. 5(2 + 13y)
11. 2x(- yz – 8)

11. -7(2x + 9)

12. 5(2y + 5x)

14. 3(x + 2y + z)

Scrambled Answers

-12x -18y

24x + 60y

-16x + 8y

25x + 10y

3x + 6y + 3z

~~8x + 24y~~ (1)

-14x – 63

6x – 6y

65y + 10

-7x + 42y

3x – 12

2x – 24

-2xyz – 16x

3x + 6

~Must show all work.~

Simplifying Algebraic Expressions: Guided Notes

Assignment: **Substitute the given numerical values to demonstrate equivalency.**

a=-1 b=-4 c=-3 k=-2 n=4 m=3 x=2 y=5

Use the distributive property to rewrite each expression.

1) 3(x + 8) x=2

3x + 24

3(2) + 24

6 + 24 = 30

2) 7(m + 6)

3) -8(b + 5)

4) -7(n + 2)

5) -4(k + 8)

6) (c – 8)(-8)

7) -5(a + 9)

8) (x – 6)(-4)

9) 2(a + b)

10) 4(x – y)

11) 3(2y + 1)

12) -4(3x + 5)

Simplify each expression. Make it distributive, first. Find a common factor.

13. 4y + 7y y=5

y(4+7)

y(11) = 11y = 11(5) = 55

14) 10k – k

15) 2m + 5 – 8m

16) n + 5n

17) 5x + 4 + 9x

18) 3r + 7 – 3r

Module 3 Lesson 4 Review Part 1

For each expression, write each sum as a product of two factors. Use the distributive property.

Questions:

1. $4 \cdot 7 + 4 \cdot 9$
2. $x \cdot 10 + 7 \cdot 10$
3. $x \cdot 7 + y \cdot 7$
4. $(3 + 2) + (3 + 2)$
5. $(x + 5) + (x + 5) + (x + 5)$
6. $(x + y) + (x + y) + (x + y) + (x + y)$
7. $2x + (2 + x) + 2 \cdot 2$
8. $5x + (y + x) + 5y$
9. $6 \cdot 2 + 6 \cdot 11$
10. $x \cdot 8 + 7 \cdot 8$
11. $x \cdot 10 + y \cdot 10$
12. $(1 + 3) + (1 + 3) + (1 + 3)$
13. $(x + y) + (x + y)$
14. $3x + (1 + x) + 3 \cdot 1$
15. $7x + (y + x) + 7y$

Scrambled Answers:

- $10(x + 7)$
- $8(x + y)$
- $10(x + y)$
- $6(x + y)$
- $4(7 + 9)$
- $3(x + 2)$
- $3(x + 5)$
- $7(x + y)$
- $6(2 + 11)$
- $4(x + 1)$
- $3(1 + 3)$
- $2(x + y)$
- $8(x + 7)$
- $2(3 + 2)$
- $4(x + y)$

Notes

Combining Like Terms

One way to simplify an expression is to "combine like terms."

What does it mean to combine like terms?	To add or subtract terms that have the same variables raised to the same exponents.

You can only combine terms that have the same variables *and the same* exponents.

*To combine like terms, **first** use the commutative property to move all like terms together. **Then**, combine the coefficients of the variables.*		
Example 1:	*Example 1:*	*Example 1:*
$2a + 3b - 4a$ $2a - 4a + 3b$ $-2a + 3b$ Note: Make sure to move any negative signs with the term it is before!	$14m - 3n^2 - 2n^2 + 3m$	$5x + 4x - 6 + 5x^2$

Note: all of your answers should be arranged so that the variables are in ______________ order first, then in order from greatest to least ______________________.

Watch out for the following common mistakes! Circle the mistakes below:		
Mistake #1:	*Mistake #2:*	*Mistake #3:*
$a^2 - 4a + 5a$ $2a^2$ *You can **ONLY** combine terms when the variable has the same exponent.*	$3y + 4x^2 - 3y + 5y$ $3y - y + 5y + 4x^2$ $7y + 4x^2$ *You should **ALWAYS** put the variables of your answer in alphabetical order, then in order by exponent.*	$3h + 14g - 5h + 5g$ $3h + 5h + 14g - 5g$ $8h + 9g$ $9g + 8h$ *You should **ALWAYS** move the negative sign along with the term that is after it.*

Practice

Combining Like Terms

Which terms are like terms? (Not all terms will be used.)

Circle all terms that can be combined with 3a.	*Draw a square around all terms that can be combined with 4b.*	*Underline all terms that can be combined with a^2.*	*Draw an X through all terms that can be combined with 5.*

1. $14a$
2. $5ab$
3. $3b$
4. $3a^2$
5. $4b^2$
6. 17
7. 100
8. $14ab$
9. $5a^3$
10. $4a$
11. $16b$
12. $73a^2$

Simplify the following expressions by combining like terms. Show all work on a separate sheet of paper and box your answer.

13. $4x - 6x$
14. $7y + 5y - 5y$
15. $4r + 4y - 8$
16. $3m + 4n - 6n$
17. $4g + 6g - 3g$
18. $15f - 5 + 2f$
19. $13x - 7y + 4x$
20. $5x^2 - 4x + 9x^2$
21. $4b + 7a - 8$
22. $13r + 5s - 2r$
23. $a + a + 3b + b$
24. $3y - 4y^2 + 3y$
25. $(3a - b) + 2a$
26. $2w + 4w^2 - 5w^3$
27. $c^3 + 4c - 4c^3$
28. $a - 3b + 5c + 4a$
29. $2x + 7x - 6x + 8$
30. $11q + 5p - 9q + 7p$
31. $3mn + 4m - 2mn$
32. $0t - 9t + 6u + 4u^5$
33. $11d + 5f - 21d + 5 - 8$
34. $12 + 9x - 6x - 19$
35. $y^2 + 3y^2 - 6y + 4y^2$
36. $2 - 5t + 8 + 5t - 8$

When part of an expression is over or under a division bar, you must act as if that part of the expression is inside of parenthesis. Use PEMDAS to decide if you can simplify the expression any further. (Think: did you get a fraction that you can simplify?)

37. $\dfrac{14r + 12s}{4s - 10s}$
38. $\dfrac{3x^2}{12 - 14x^2}$
39. $\dfrac{2 - 5t}{2 + 5t - 4t}$
40. $\dfrac{2x - 6y + 4x}{3y - 8 + y}$
41. $\dfrac{11d + 9d}{8d - 3d}$
42. $\dfrac{12x - 7x}{5x}$

Bonus: Simplify the expression below by combining like terms.

$$4z + x - 5x + 7y - 3x + 5y^2 - 3z + 16z + 14x - 5$$

Practice

Combining Like Terms Puzzle

Simplify each expression by combining like terms. Find the answer at the bottom of the page. Then write the letter on the appropriate line below to spell out a secret message. (Some letters may be used more than once!)

Did you hear the one about the acupuncture?

___ ___   ___ ___ ___   ___   ___ ___ ___
1 2   3 4 5   6   7 8 9

___ ___ ___ ___   ___ ___ ___ ___ !
10 11 12 13   14 15 16 17

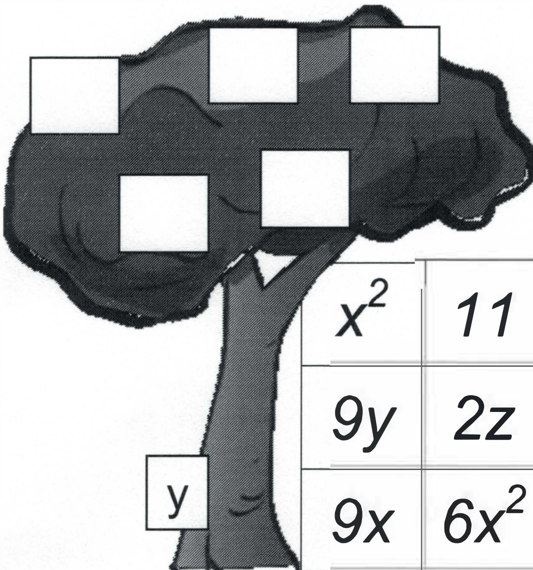
1. $2m + 3m^2 - 4m$	2. $2x + x – 4y$	3. $2m + 4m – 3m^2$	4. $2y + 14x – 7x + 9y$
5. $8n – 4n^2 + 8n$	6. $11g – 9g + 8g$	7. $3m^2 – 2m + 4m$	8. $20 + 10q + 3q – 4$
9. $4xy + x + 2xy$	10. $6m^2 + 6m – 9m^2$	11. $3n – 6mn + 2n$	12. $\frac{3}{2}x – y + \frac{1}{2}x + 3y$
13. $y + x + y + x$	14. $8n + 4n^2 – 8n$	15. $5 + 5mn – 11mn$	16. $15y + 6y – 3x + xy$
17. $3xy – 5xy + 21y$			

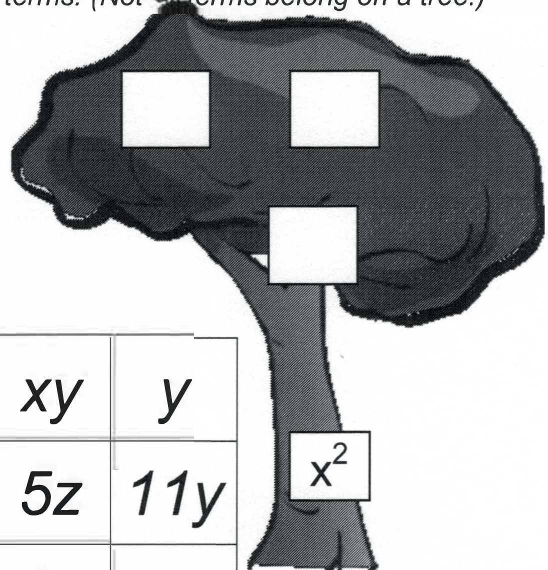
I. $3m^2 – 2m$	**N. $-3x + xy + 21y$**	**A. $7x + 11y$**
S. $-4n^2 + 16n$	**W. $-3m^2 + 6m$**	**E. $-6mn + 5n$**
E. $–2xy + 21y$	**J. $3m^2 + 2m$**	**A. $13q + 16$**
O. $-6mn + 5$	**B. $x + 6xy$**	**L. $2x + 2y$**
T. $3x – 4y$	**A. $10g$**	**D. $4n^2$**

Enrichment Activity 1

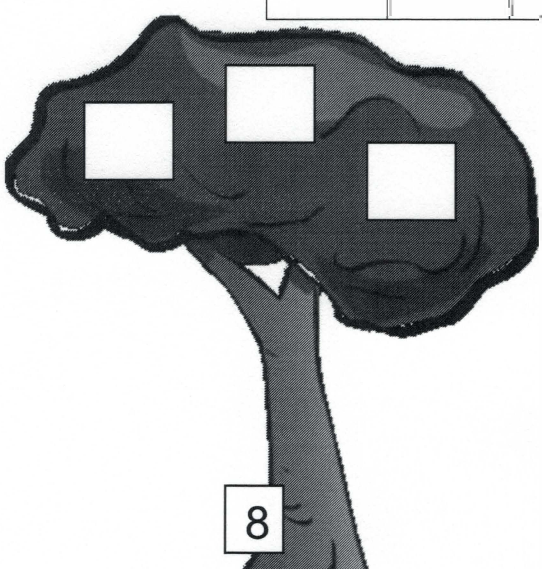
Combining Like Terms

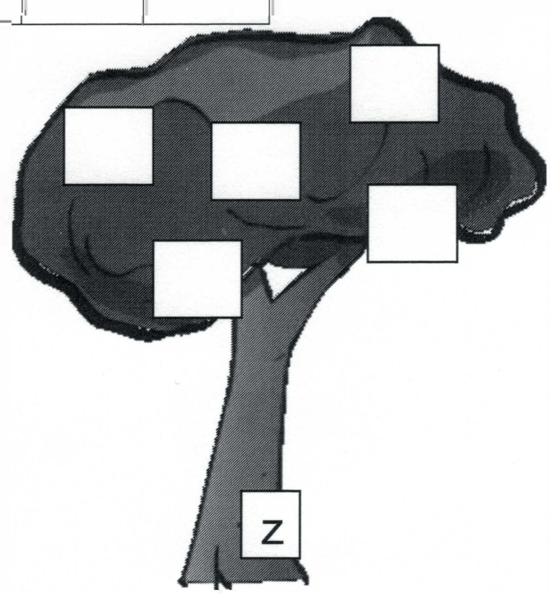
Have you ever heard the phrase "you can't compare apples and oranges?" Place each of the terms below on the proper "tree" that contains like terms. (Not all terms belong on a tree!)





x^2	11	$4z$	xy	y
$9y$	$2z$	$2y^3$	$5z$	$11y$
$9x$	$6x^2$	$9y$	$2z$	19
7	$-x^2$	$3y$	$2x$	$7z$





Lesson 23: Solve Equations with Distributive Property

Lesson 8: Solving Equations with the Distributive Property & Combining Like Terms

Directions: Solve each equation. Use algebraic steps and check the solutions.

1. $5(h + 10) = 70$

2. $3(z + 8) = 42$

3. $3x + 2(x + 4) = 23$

4. $5m + 4(m + 6) = 33$

5. $a + 3a + (a + 4) = 39$

6. $c + 4c + (c + 8) = 20$

7. $-2(5 + 6m) + 16 = -90$

8. $22 - 5(6v - 1) = -63$

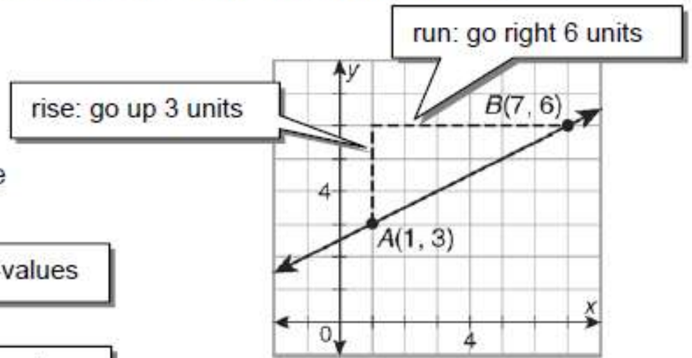
LESSON 3-5

Reteach

Slopes of Lines

The **slope** of a line describes how steep the line is. You can find the slope by writing the ratio of the **rise** to the **run.**

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{3}{6} = \frac{1}{2}$$



You can use a formula to calculate the slope m of the line through points (x_1, y_1) and (x_2, y_2).

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Change in y-values

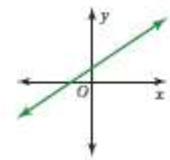
Change in x-values

To find the slope of $\overline{AB}$ using the formula, substitute (1, 3) for (x_1, y_1) and (7, 6) for (x_2, y_2).

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \frac{6 - 3}{7 - 1}$	Substitution
$= \frac{3}{6}$	Simplify.
$= \frac{1}{2}$	Simplify.

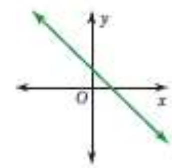
Slope

Positive Slope



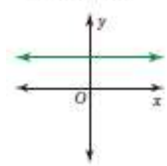
The line rises from left to right.

Negative Slope



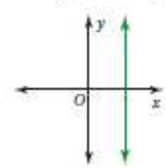
The line falls from left to right.

Slope of 0



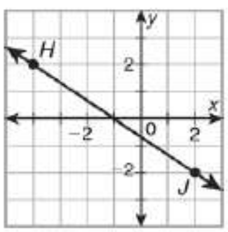
The line is horizontal.

Undefined Slope



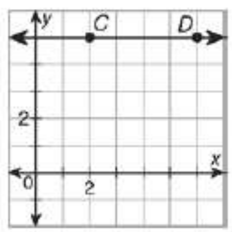
The line is vertical.

Use the slope formula to determine the slope of each line.



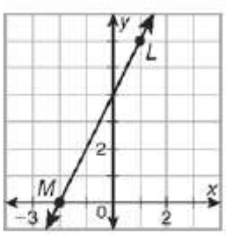
1. $\overleftrightarrow{HJ}$

____



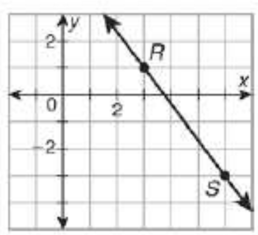
2. $\overleftrightarrow{CD}$

____



3. $\overleftrightarrow{LM}$

____



4. $\overleftrightarrow{RS}$

____

Slippery Slopes

Name________________________________

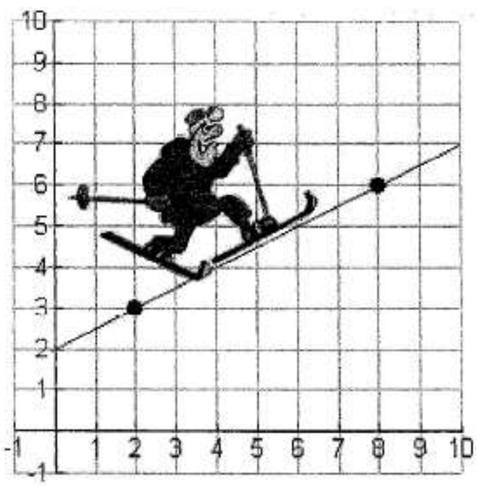
Directions: (a) Write the coordinates of the two points shown on the following graphs.
(b) Find the slope, *m*, of the line connecting the two points.

$m = \frac{Y_2 - Y_1}{X_2 - X_1}$

$x_1\ y_1$ $x_2\ y_2$ $\frac{\text{rise}}{\text{run}}$

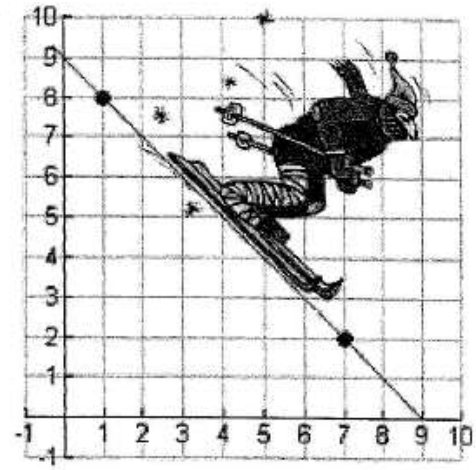
1. Points: (_,_) (_,_)

m = __________________



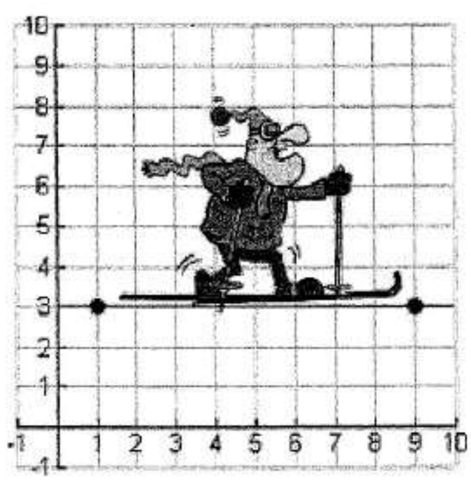
2. Points: _______ _______

m = __________________



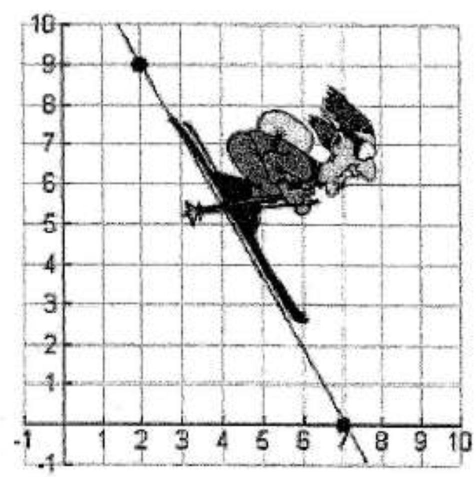
3. Points: _______ _______

m = __________________



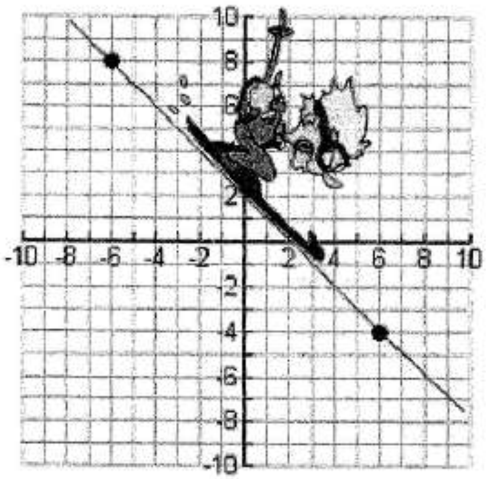
4. Points: _______ _______

m = __________________



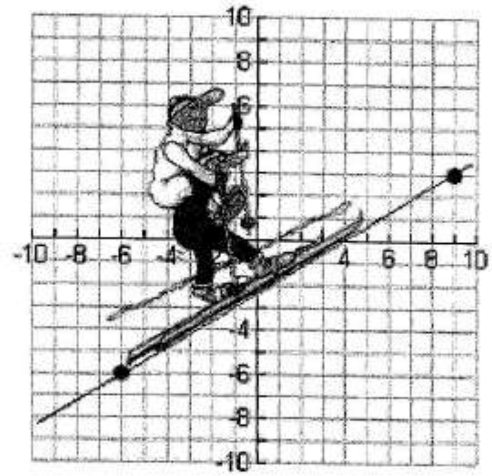
5. Points: _______ _______

$m =$ ____________________



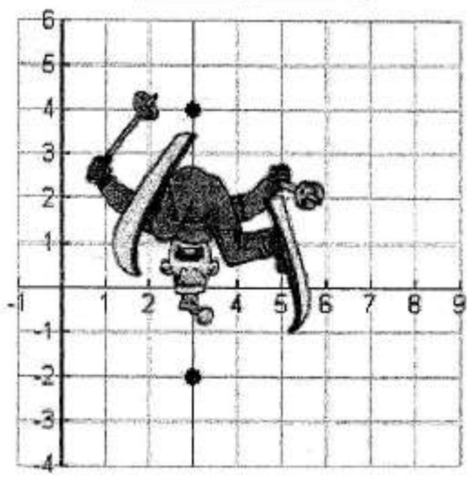
6. Points: _______ _______

$m =$ ____________________



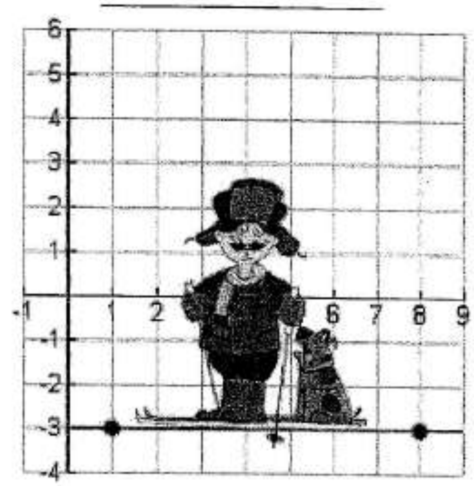
7. Points: _______ _______

$m =$ ____________________



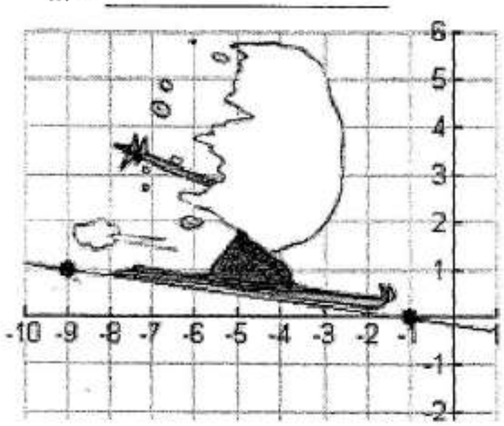
8. Points: _______ _______

$m =$ ____________________



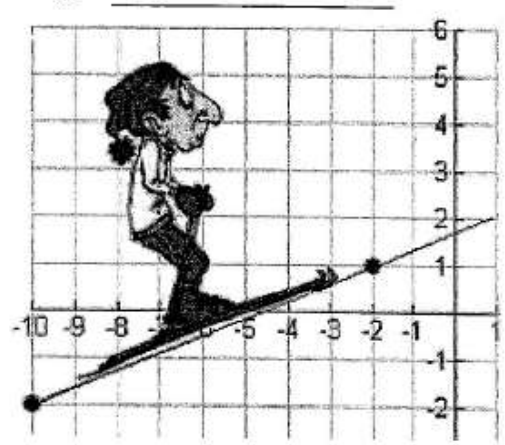
9. Points: _______ _______

$m =$ ____________________

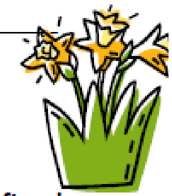


10. Points: _______ _______

$m =$ ____________________

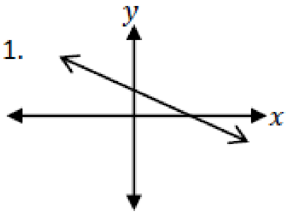


Name ______________________________________ Period ______ Date ______________



PRACTICE – Everything Slope So Far

Examine the graphs below. State whether the slope of the line would be positive, negative, 0 or undefined.

1. 

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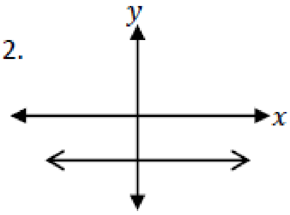
2. 

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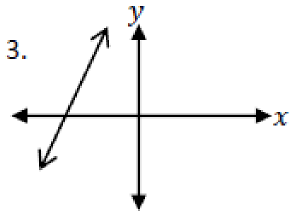
3. 

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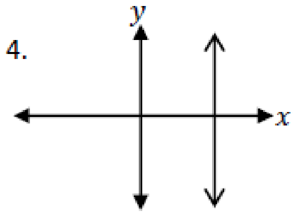
4. 

____

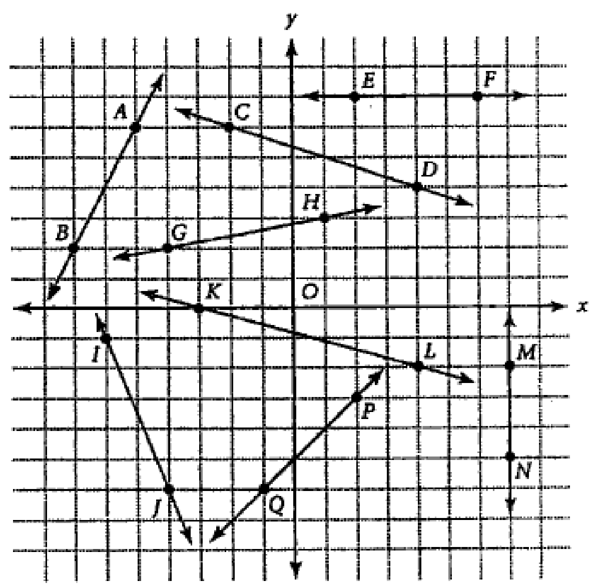
Use the graph to find the slope of each line.

5. $\overleftrightarrow{AB}$ __________ 6. $\overleftrightarrow{CD}$ __________

7. $\overleftrightarrow{EF}$ __________ 8. $\overleftrightarrow{GH}$ __________

9. $\overleftrightarrow{IJ}$ __________ 10. $\overleftrightarrow{KL}$ __________

11. $\overleftrightarrow{MN}$ __________ 12. $\overleftrightarrow{PQ}$ __________



Find the slope of each line.

13. rise: −5; run: −5 __________

14. rise: 2; run: 3 __________

15. rise: −3; run: 4 __________

16. rise: −2; run: −5 __________

Simplify these fractions.

17. $\frac{5}{0} =$ 18. $\frac{0}{-2} =$ 19. $\frac{3}{-1} =$ 20. $\frac{-1}{-2} =$

21. $\frac{-2}{6} =$ 22. $\frac{0}{4} =$ 23. $\frac{-5}{-1} =$ 24. $\frac{3}{0} =$

25. All vertical lines have a slope of ______________________.

26. All horizontal lines have a slope of ______________________.

27. Slope is represented by the variable _______.

Exponents

1) What is an exponent?

Exponent is a shortcut for showing repeated multiplication.

For example, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

2 is called the base and 5 is called the exponent.

The "base" is the number being multiplied, the "exponent" indicates how many times to multiply the base.

2^5 can be read as "2 raised to the fifth power" or "2 raise to the power of 5" or "2 raised by the exponent of 5" or "2 to the 5th".

If the exponent is 0 then the answer is always 1.

Example: $5^0 = 1$, $32^0 = 1$

Name the base and exponent of each expression.

2) Example: 12^4
Base = 12
Exponent = 4

3) 5^8
Base = _______

Exponent = ________

4) $(-3)^4$
Base = _______

Exponent = ________

5) $\left(\frac{1}{4}\right)^5$
Base = _______

Exponent = ________

6) x^6
Base = _______

Exponent = ________

7) n^{13}
Base = _______

Exponent = ________

Lesson 26: Exponents

Wrtie each exponential expression as a repeated multiplication problem

8) Example: $3^6 = 3 \text{ x } 3 \text{ x } 3 \text{ x } 3 \text{ x } 3 \text{ x } 3$

9) Example: $m^3 = \text{m x m x m}$
NOTE: Since x for multiply looks like the letter x, practice using a raised dot or ( ) to show multiplication.

So, m^3 should look like $m \cdot m \cdot m$

10) $2^5 =$ ______________________________

11) $(-3)^3 =$ ______________________________

12) $5^2 =$ ______________________________

13) $\left(\frac{1}{3}\right)^4 =$ ______________________________

14) $x^5 =$ ______________________________

15) $n^6 =$ ______________________________

Evaluate the exponent.

16) Example: $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

17) $2^5 =$

18) $1^7 =$

19) $(-2)^4 =$

20) $4^3 =$

21) $9^2 =$

22) $6^1 =$

23) $(-3)^3 =$

24) $7^0 =$

25) $\left(\frac{1}{2}\right)^2 =$

Lesson 26: Exponents

Write the repeated multiplication as an Exponent

26) Example: $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

The 3 is the number being multiplied so it is the BASE

The 3 is being multiplied seven times, so 7 is going to be the EXPONENT

Result: 3^7

27) Example: $x \cdot x \cdot x \cdot x \cdot x$

The x is the number being multiplied so it is the BASE

The x is being multiplied five times, so 5 is going to be the EXPONENT

Result: x^5

28) $-5 \cdot -5 \cdot -5 \cdot -5 \cdot -5 \cdot -5 =$ ________

29) $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} =$ ________

30) $y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y =$ ________

31) $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 =$ ________

Exponent Rules 1: Product of Powers and Quotient of Powers

Product of Powers Rule: Keep the base and add the exponents

1) Product of Powers Rule. When multiplying two bases of the same value, keep the bases the same and then add the exponents together to get the solution.

Ex: $2^3 \cdot 2^4$ Since 2^3 is really this: $2 \cdot 2 \cdot 2$ and 2^4 is really this: $2 \cdot 2 \cdot 2 \cdot 2$ when you multiply the two expressions together you will get this: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ which will end up being 2^7

So, $2^3 \cdot 2^4 = 2^7$ as you can see, the base stayed the same and you simply add the exponents.

Ex: $(-3)^5 \cdot (-3)^6 = (-3)^{5+6} = (-3)^{11}$

Ex: $m^5 m^3 m^8 = m^{5+3+8} = m^{16}$

Use the Product of Powers Rule to simplify

2) $5^7 \cdot 5^2 =$ ________

3) $(-2)^4 \cdot (-2)^6 =$ ________

4) $v^3 v^5 =$ ________

5) $\left(\frac{4}{5}\right)^8 \cdot \left(\frac{4}{5}\right)^4 =$ ________

6) $4^5 \cdot 4^8 \cdot 4^2 =$ ________

7) $(-3)^2 \cdot (-3)^5 =$ ________

8) $9^0 \cdot 9^4 =$ ________

9) $(-7)^0 \cdot (-7)^0 =$ ________

Quotient of Powers Rule: Keep the base and SUBTRACT the exponents.

10) Quotient of Powers Rule: When dividing two bases that are the same value, keep the bases the same and then SUBTRACT the exponents to get the solution.

Ex: $\frac{2^7}{2^3} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$ if you cancel out a common factors from the top and bottom,

you can see that there are three 2's on the top that will cancel with the three 2's on the bottom.

You will get $2 \cdot 2 \cdot 2 \cdot 2$ which simplifies to 2^4.

So, $\frac{2^7}{2^3}$ is really 2^{7-3} which simplifies to 2^4. .So keep the base and subtract the exponents.

Ex: $\frac{(-7)^{12}}{(-7)^4} = (-7)^{12-4} = (-7)^8$

Ex: $\frac{m^9}{m^3} = m^{9-3} = m^6$

Use the Quotient of Powers Rule to simplify

11) $\frac{6^{13}}{6^{12}}$

12) $\frac{(-1)^8}{(-1)^3}$

13) $\frac{7^{14}}{7^{14}}$

14) $\frac{12^{15}}{12^4}$

15) $\frac{k^8}{k^2}$

16) $\frac{y^{15}}{y}$

Exponent Rules 2: Power to a Power, Product to a Power and Quotient to a Power

Power to a Power Rule: When a exponent is raised to an exponent, keep the base and MULTIPLY the exponents.

1) EX: $(3^2)^4$ The base is now 3^2 and the exponent is 4.

If we expanded this it would look like : $3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$

If we applied the Product of Powers Rule, we would keep the 3 and ADD the exponents and get:

$3^{2+2+2+2} = 3^8$ Well....2 + 2 + 2 + 2 is the same as 2 x 4.

So the rule states that we keep the 3 and multiply the two exponents.

$(3^2)^4 = 3^{2 \cdot 4} = 2^8$

Ex: $((-5)^3)^4 = (-5)^{3 \cdot 4} = (-5)^{12}$

Ex: $(x^6)^4 = x^{6 \cdot 4} = x^{24}$

Ex: $(5^8)^0 = 5^{8 \cdot 0} = 5^0 = 1$

Use the Power to a Power Rule to simplify

2) Example: $(4^5)^3 = 4^{5 \cdot 3} = 4^{15}$

3) $((-2)^4)^3$ = ________

4) $(5^6)^2$ = ________

5) $(y^7)^3$ = ________

6) $((-8)^0)^3$ = ________

7) $(9^4)^0$ = ________

8) $(g^3)^5$ = ________

9) $(1^6)^7$ = ________

Lesson 28: Exponent Properties 2 - Operations to a Power

Product to a Power Rule: Distribute the exponent to each factor of the product

10) Ex: $(3 \cdot 2)^4$ in this problem the base is $(3 \cdot 2)$ and the exponent is 4.

The expanded form would look like: $(3 \cdot 2)(3 \cdot 2)(3 \cdot 2)(3 \cdot 2)$

Using the commutative and associative properties we can rearrange it to look like this:

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Now to simplify using exponents it would look like:

$3^4 \cdot 2^4$

So, the Product to a Power Rule states that we distribute the exponent to each factor of the base. You can see that the exponent of 4 was distributed to both the 3 and the 2.

$(3 \cdot 2)^4 = 3^4 \cdot 2^4$

Ex: $(5x)^3$: The exponent is 3 and the factors of the base are 5 and x, so distribute the 3 to both the 5 and x to get

Answer: $(5x)^3 = 5^3 \cdot x^3$

Use the Product to a Power Rule to simplify

11) Ex: $(9 \cdot 3)^4 = 9^4 \cdot 3^4$

12) $(4 \cdot 6)^5 =$ ___________

13) $(-3 \cdot 6)^4 =$ ___________

14) $(12 \cdot 8)^3 =$ ___________

15) $(5 \cdot 2)^0 =$ ___________

16) $(3m)^6 =$ ___________

17) $(5n)^7 =$ ___________

18) $(8xy)^{12} =$ ___________

Lesson 28: Exponent Properties 2 - Operations to a Power

Quotient to a Power Rule: Distribute the exponent to the numerator and denominator

19) Example: $\left(\frac{2}{3}\right)^4$ here the base is $\frac{2}{3}$ and the exponent is 4.

If we were to expand it it would look like:

$$\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}=\frac{2\cdot 2\cdot 2\cdot 2}{3\cdot 3\cdot 3\cdot 3}=\frac{2^4}{3^4}$$

So, you can see that the exponent of 4 was distributed to both the 2 in the numerator and the 3 in the denominator.

Answer: $\left(\frac{2}{3}\right)^4=\frac{2^4}{3^4}$

Ex: $\left(\frac{x}{5}\right)^6=\frac{x^6}{5^6}$

20) $\left(\frac{3}{5}\right)^4=$ ________

21) $\left(\frac{1}{2}\right)^7=$ ________

22) $\left(\frac{x}{-4}\right)^2=$ ___

23) $\left(\frac{m}{w}\right)^8=$ ________

24) $\left(\frac{9}{m}\right)^3=$ ___

25) $\left(\frac{3}{14}\right)^0=$ ________

PSAT Practice Test

3 3

Math Test – No Calculator

20 MINUTES, 13 QUESTIONS

Turn to Section 3 of your answer sheet to answer the questions in this section.

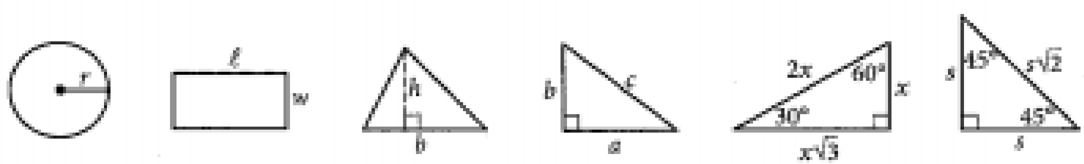
DIRECTIONS

For questions 1-10, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 11-13**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 11 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

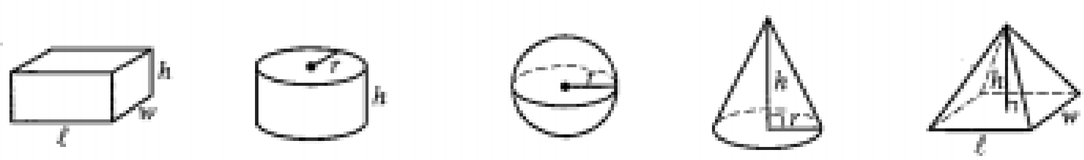
NOTES

1. The use of a calculator **is not permitted**.
2. All variables and expressions used represent real numbers unless otherwise indicated.
3. Figures provided in this test are drawn to scale unless otherwise indicated.
4. All figures lie in a plane unless otherwise indicated.
5. Unless otherwise indicated, the domain of a given function f is the set of all real numbers x for which $f(x)$ is a real number.

REFERENCE



$A = \pi r^2$, $C = 2\pi r$ | $A = \ell w$ | $A = \frac{1}{2}bh$ | $c^2 = a^2 + b^2$ | Special Right Triangles



$V = \ell wh$ | $V = \pi r^2 h$ | $V = \frac{4}{3}\pi r^3$ | $V = \frac{1}{3}\pi r^2 h$ | $V = \frac{1}{3}\ell wh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

CONTINUE

1

$$0,\ 1\frac{1}{2},\ 3,\ 4\frac{1}{2},\ x,\ \ldots$$

In the sequence above, the first term is 0. Each term after the first is $\frac{3}{2}$ greater than the term before it. What is the value of x ?

A) 7

B) 6

C) $5\frac{1}{2}$

D) 5

2

$$\frac{4x}{5} = 20$$

In the equation above, what is the value of x ?

A) 25
B) 24
C) 16
D) 15

3

Angela is playing a video game. In this game, players can score points only by collecting coins and stars. Each coin is worth c points, and each star is worth s points.

- The first time she played, Angela scored 700 points. She collected 20 coins and 10 stars.
- The second time she played, Angela scored 850 points. She collected 25 coins and 12 stars.

Which system of equations can be used to correctly determine the values of c and s ?

A) $10c + 20s = 700$
$12c + 25s = 850$

B) $20c + 10s = 700$
$25c + 12s = 850$

C) $20c + 700s = 10$
$25c + 850s = 12$

D) $700c + 20s = 10$
$850c + 25s = 12$

4

$$3x + y = 24$$
$$x^2 + y = 64$$

Which of the following ordered pairs (x, y) is a solution to the system of equations above?

A) $(0, 24)$
B) $(0, 64)$
C) $(8, 0)$
D) $(-8, 0)$

5

Marisol drove 3 hours from City A to City B. The equation below estimates the distance d, in miles, Marisol traveled after driving for t hours.

$$d = 45t$$

Which of the following does 45 represent in the equation?

A) Marisol took 45 trips from City A to City B.

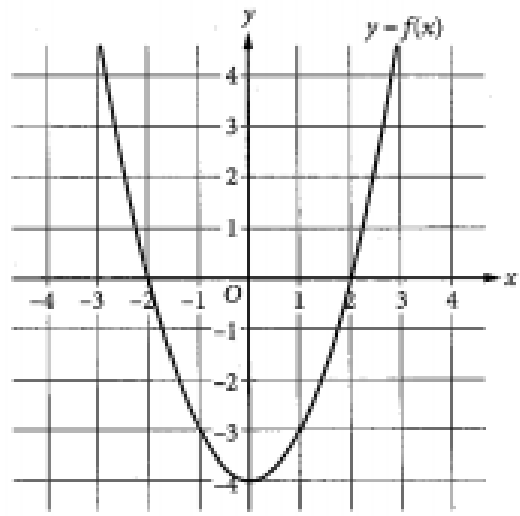
B) The distance between City A and City B is 45 miles.

C) Marisol drove at an average speed of about 45 miles per hour.

D) It took Marisol 45 hours to drive from City A to City B.

6

The graph of a function f is shown in the xy-plane below.



Which of the following equations could represent f?

A) $f(x) = x^2 - 4$

B) $f(x) = x^2 - 2$

C) $f(x) = x^2 + 2$

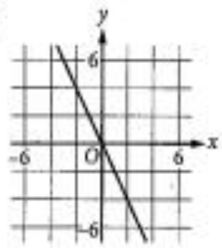
D) $f(x) = x^2 + 4$

CONTINUE

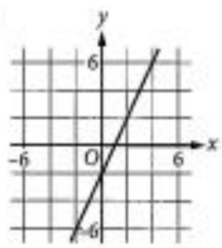
7

Which of the following is the graph of $y = \frac{1}{2}x - 2$ in the xy-plane?

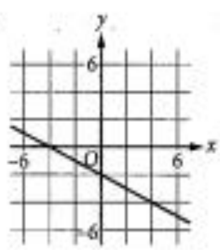
A)



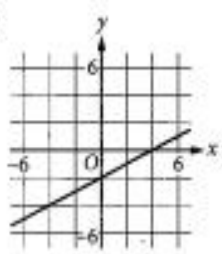
B)



C)



D)



8

$$y = 2x - 3$$
$$3y = 5x$$

In the solution to the system of equations above, what is the value of y ?

A) -15

B) -9

C) 9

D) 15

CONTINUE

9

$$(2h-3)-(h^2-5h-8)$$

Which of the following expressions is equivalent to the expression shown above?

A) h^2+3h+5

B) $h^2-3h-11$

C) $-h^2+7h+5$

D) $-h^2-3h-11$

10

Which of the following is equivalent to the expression x^2-8x-9 ?

A) $(x-3)^2$

B) $(x-3)(x-6)$

C) $(x+9)(x-1)$

D) $(x-9)(x+1)$

CONTINUE

3 3

11

$$47 = 4z - 11$$

What is the value of z that satisfies the equation above?

12

Line s is drawn in the xy-plane and has an equation $3y - 6x = 9$. Line t is parallel to line s. What is the slope of line t ?

13

$$6 = x(1 + 2x)$$

If x is a solution to the equation above and $x > 0$, what is the value of x ?

STOP

If you finish before time is called, you may check your work on this section only.

Do not turn to any other section.

Math Test – Calculator

40 MINUTES, 25 QUESTIONS

Turn to Section 4 of your answer sheet to answer the questions in this section.

DIRECTIONS

For questions 1-21, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 22-25**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 22 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

NOTES

1. The use of a calculator **is permitted.**
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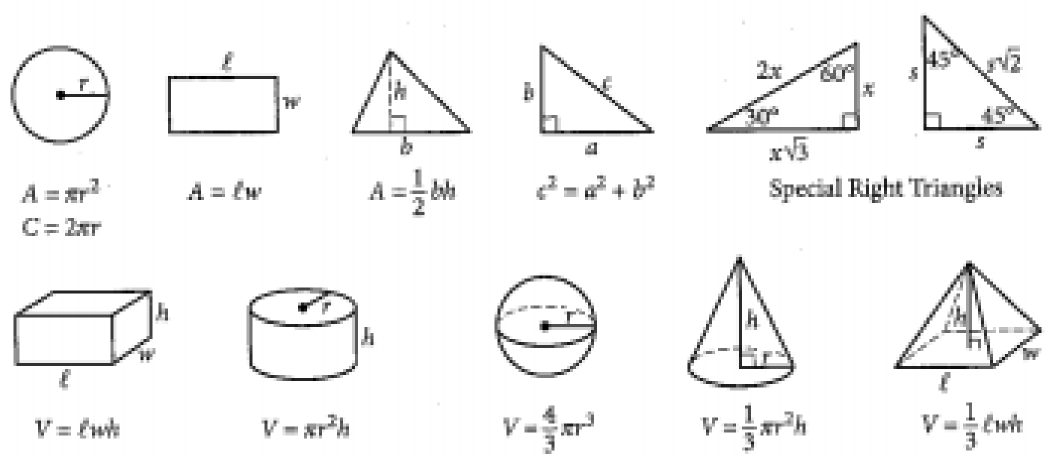
4. All figures lie in a plane unless otherwise indicated.
5. Unless otherwise indicated, the domain of a given function f is the set of all real numbers x for which $f(x)$ is a real number.

REFERENCE



The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

CONTINUE

1

On a floor plan for Rosedale Middle School, 1 inch represents 10 feet. If Sarah's classroom is 2 inches by 3 inches on the floor plan, what are the actual dimensions of her classroom?

A) 20 feet by 30 feet

B) 40 feet by 60 feet

C) 200 feet by 300 feet

D) 20 yards by 30 yards

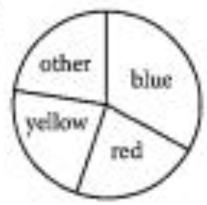
2

Color Survey

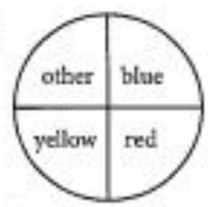
Favorite color	Number of students
blue	8
red	6
yellow	6
other	4

Each of the 24 children in Mr. Ishibashi's kindergarten class was asked, "What is your favorite color?" The results are shown in the table above. Which of the following circle graphs represents the information in the table?

A)


B)


C)


D)


3

For a school fund-raiser, 10 students sold a total of 90 boxes of cookies. Which of the following can be calculated from this information?

A) The average number of boxes sold per student
B) The median number of boxes sold per student
C) The greatest number of boxes sold by one student
D) The least number of boxes sold by one student

4

Cathy has n CDs. Gerry has 3 more than twice the number of CDs that Cathy has. In terms of n, how many CDs does Gerry have?

A) $3n - 2$
B) $3n + 2$
C) $2n - 3$
D) $2n + 3$

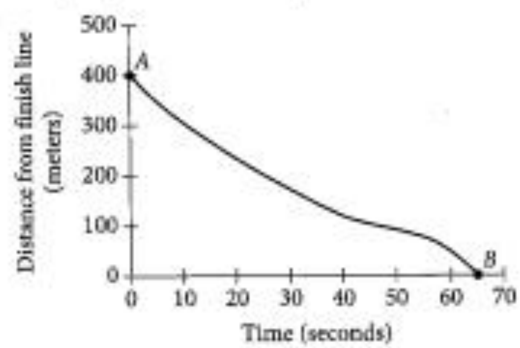
5

A librarian has 43 books to distribute to a group of children. If he gives each child 2 books, he will have 7 books left over. How many children are in the group?

A) 15
B) 18
C) 25
D) 29

6

The graph below shows the relationship between time, in seconds, and distance from the finish line, in meters, for Juan while he competed in a race.



Which of the following is the correct interpretation of point B in the context of this problem?

A) At 65 seconds, Juan was at the finish line.
B) Juan's average speed was slightly over 6 meters per second.
C) Juan's fastest speed during the race was approximately 7 meters per second.
D) Juan was 400 meters away from the finish line when the time was equal to zero.

7

During migration, a gray whale swam 5,040 miles. The whale swam a total of 672 miles during its first 7 days of migration. If the whale swam at the same rate during the entire migration, how many miles did it swim during the first 30 days of migration?

A) 168
B) 695
C) 1,176
D) 2,880

8

If $3x - 6 = 21$, what is the value of $x - 2$?

A) 3
B) 5
C) 7
D) 11

Questions 9 and 10 refer to the following information.

A tree is planted and is expected to grow according to the model below, where t is the number of years since the tree was planted and H is the height of the tree, in feet.

$$H = 3t + 5$$
$$0 \leq H \leq 100$$

9

How many years after the tree is planted does the model predict the tree will reach a height of 65 feet?

A) 200
B) 23
C) 20
D) 17

10

According to the model, which of the following statements is true?

A) The tree was 3 feet tall when planted.
B) The tree is expected to increase in height at a rate of 3 feet per year.
C) The tree is expected to increase in height at a rate of 1 foot every 3 years.
D) The tree is expected to reach a maximum height of 3 feet.

CONTINUE

4 **4**

11

Geoff earns \$9.25 per hour before taxes, and he works 40 hours per week. Geoff's employer withholds (takes out) 10.65% of Geoff's income for taxes. What are Geoff's total weekly earnings after taxes are withheld?

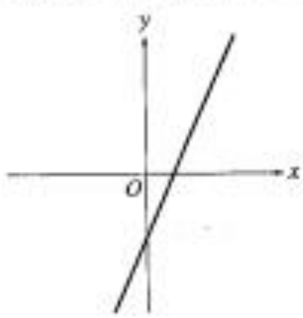
A) \$39.41

B) \$59.90

C) \$330.60

D) \$370.00

12



Which of the following could be an equation of the line graphed in the *xy*-plane above?

A) $y = 2x - 6$

B) $y = 2x + 3$

C) $y = -2x - 6$

D) $y = -2x + 3$

13

In the equation below, c is a constant.

$$(x + c)^2 = 25$$

For which of the following values of c is $x = -2$ a solution to the equation?

A) -7

B) -3

C) 2

D) 10

14

A certain forest is 253 acres. To estimate the number of trees in the forest, a ranger randomly selects 5 different 1-acre parcels in the forest and determines the number of trees in each parcel. The numbers of trees in the sample acres are 51, 59, 45, 52, and 73. Based on the mean of the sample, which of the following ranges contains the best estimate for the number of trees in the entire forest?

A) 11,000 to 12,000

B) 12,500 to 13,500

C) 13,500 to 14,500

D) 18,000 to 19,000

CONTINUE

15

This year, a town has a budget of $1,000,000. Next year, the budget will increase by 2.5%. What will the town's budget be next year?

A) $1,025,000

B) $1,250,000

C) $2,050,000

D) $2,500,000

16

The table below shows the results of a survey of high school students who plan to attend college. It shows whether the students plan to attend a 2-year college or a 4-year college and whether the students plan to attend an in-state or an out-of-state college.

College Planning Survey

	2-year college	4-year college	Total
In-state	18	16	34
Out-of-state	4	12	16
Total	22	28	50

What percent of the students surveyed plan to attend an out-of-state 4-year college?

A) 12%

B) 24%

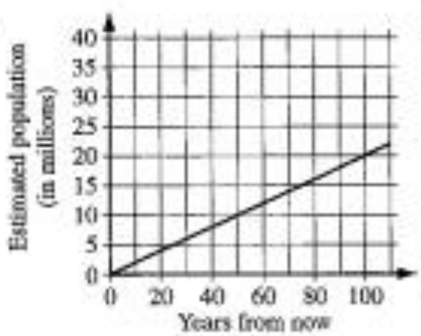
C) 43%

D) 75%

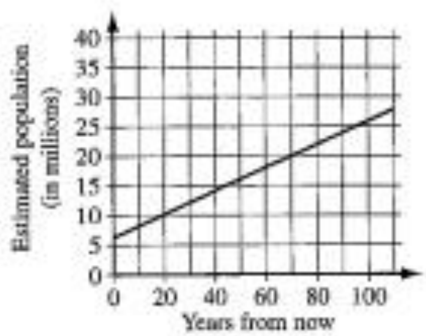
17

The current population of a country is 6.0 million people. It is estimated that the population of the country will double every 30 years. Of the following, which graph best represents the country's estimated population growth?

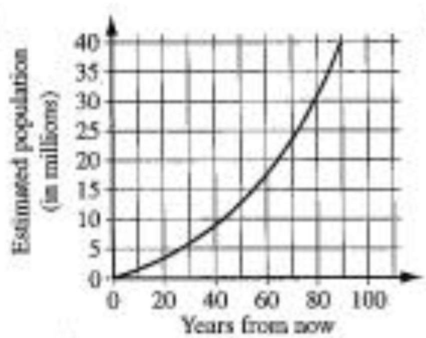
A)



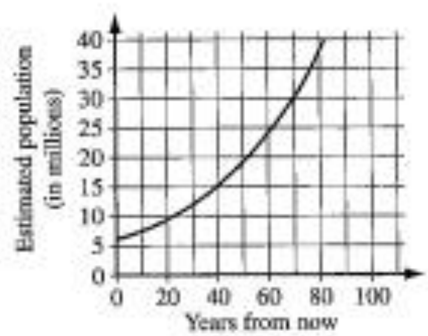
B)



C)



D)



CONTINUE

18

Megan played 5 Sudoku games that were at medium difficulty level. The mean completion time per game was 5.9 minutes. She completed a sixth Sudoku game in 2 minutes and 54 seconds. Which of the following statements must be true about the changes between Megan's completion times for the 6 games and her completion times for the 5 games?

A) The mode of the 6 completion times decreased.

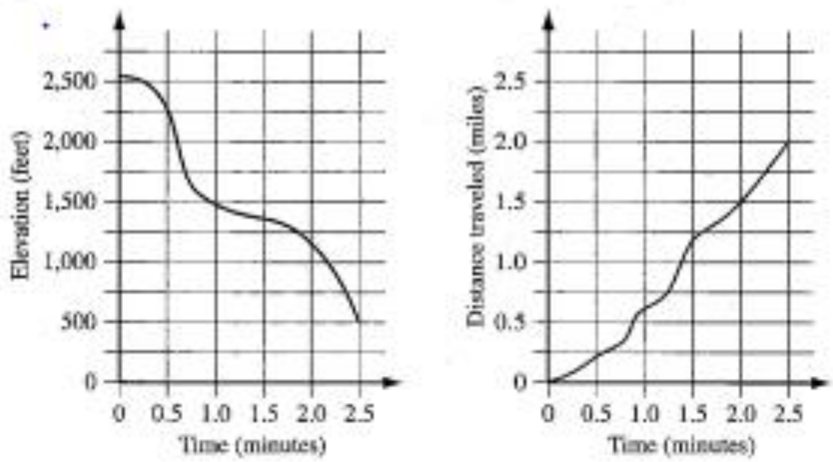
B) The median of the 6 completion times remained the same.

C) The mean completion time per game for the 6 games increased by 2.9 minutes.

D) The mean completion time per game for the 6 games decreased by 30 seconds.

19

Salma completed a downhill ski race. The graphs below represent her elevation over time and her distance traveled over time as she raced down the mountain.



Which of the following is closest to the total distance Salma had traveled when she reached an elevation of 1,500 feet?

A) 0.6 mile

B) 0.9 mile

C) 1.1 miles

D) 1.5 miles

20

A gym offers only a cycling class and a yoga class at the same time on Saturday mornings. The fitness director at the gym kept track of the number of males and females in these two classes last Saturday morning. The data are displayed in the table below.

Saturday Morning Classes

	Males	Females
Yoga	18	23
Cycling	21	17

To the nearest percent, what percent of the people who attended classes that morning were males in the yoga class?

A) 18%

B) 23%

C) 44%

D) 46%

21

Gold is often used with other metals, such as copper and zinc, to make jewelry. Approximately 58% of the mass of a certain 40-gram necklace is gold. The density of gold is 19.3 grams per cubic centimeter. What is the volume of gold, to the nearest tenth of a cubic centimeter, in the necklace?

A) 0.8

B) 1.2

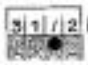
C) 4.5

D) 23.2

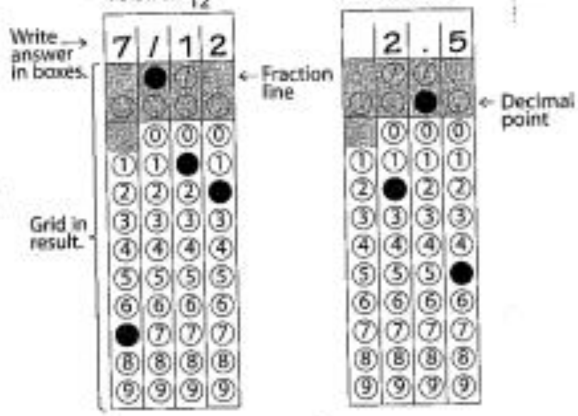
CONTINUE

DIRECTIONS

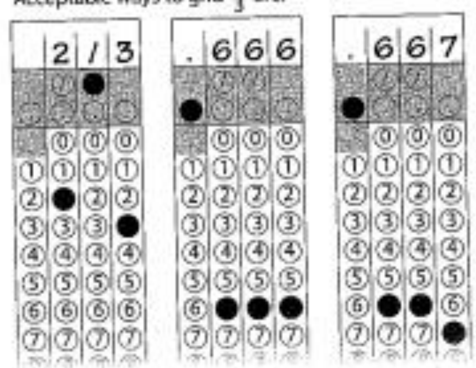
For questions 22-25, solve the problem and enter your answer in the grid, as described below, on the answer sheet.

1. Although not required, it is suggested that you write your answer in the boxes at the top of the columns to help you fill in the circles accurately. You will receive credit only if the circles are filled in correctly.
2. Mark no more than one circle in any column.
3. No question has a negative answer.
4. Some problems may have more than one correct answer. In such cases, grid only one answer.
5. **Mixed numbers** such as $3\frac{1}{2}$ must be gridded as 3.5 or 7/2. (If  is entered into the grid, it will be interpreted as $\frac{31}{2}$, not $3\frac{1}{2}$.)
6. **Decimal answers:** If you obtain a decimal answer with more digits than the grid can accommodate, it may be either rounded or truncated, but it must fill the entire grid.

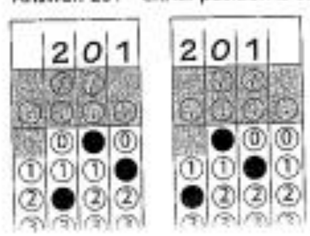
Answer: $\frac{7}{12}$ Answer: 2.5



Acceptable ways to grid $\frac{2}{3}$ are:



Answer: 201 – either position is correct



NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

22

Angelo grows vegetables and sells them at a farmers' market. The price of 2 cucumbers and 1 pepper is $3.15, and the price of 3 cucumbers and 2 peppers is $5.35. Based on this pricing, what would be the price, in dollars, of 4 cucumbers and 3 peppers? (Disregard the $ sign when gridding your answer. For example, if your answer is $1.37, grid 1.37)

23

$$2y - 2x = 8$$
$$y + 6x = 11$$

If (x, y) is the solution to the system of equations above, what is the value of $7y$?

Questions 24 and 25 refer to the following information.

A random sample of 400 town voters were asked if they plan to vote for Candidate A or Candidate B for mayor. The results were sorted by gender and are shown in the table below.

	Plan to vote for Candidate A	Plan to vote for Candidate B
Female	202	20
Male	34	144

24

The town has a total of 6000 voters. Based on the table, what is the best estimate of the number of voters who plan to vote for Candidate A?

25

The percentage of voters in the sample who answered that they are both female and planned to vote for Candidate B is $p\%$. What is the value of p ?

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section.

Fractions

Fractions are a part of a whole.

This means they are created through division.

This understanding helps us operate with fractions.

Multiplication

Multiplication is the easiest operation to do with fractions. This is because division is really a specific type of multiplication (division is defined as the inverse of multiplication - so multiplying by 1 over)

To multiply, we simply multiply numerator by numerator and denominator by denominator. This is because we can chose which order to multiply in as multiplication is *associative* and *commutative*.

$$\frac{a}{b}\bullet\frac{c}{d} = a\bullet\frac{1}{b}\bullet c\bullet\frac{1}{d}$$ dis-associating

$$= a\bullet c\bullet\frac{1}{b}\bullet\frac{1}{d}$$ commuting

$$= (a\bullet c)\bullet\left(\frac{1}{b}\bullet\frac{1}{d}\right)$$ associating

$$= ac\bullet\frac{1}{bd}$$ multiplying

$$= \frac{ac}{bd}$$ multiplying

Simplifying Fractions

Simplifying fractions comes from the understanding that fractions are formed through division. Since multiplication is associative, we can chose to do portions of the division first.

$$\frac{8}{10} = \frac{4 \bullet 2}{5 \bullet 2} = \frac{4}{5} \bullet \left(\frac{2}{2}\right) = \frac{4}{5} \bullet 1 = \frac{4}{5}$$

Here, since 2 is a factor of both 8 and 10, instead of choosing to do 4*2 and 5*2 first (what we would have to do based on order of operations - a division bar is a grouping symbol, just like parentheses), we can chose to do 2 divided by 2. This is 1. Since one is the multiplicitive identity, our result is four fifths.

This only works when we have only multiplication and division (multiplication is only associative with other multiplication not addition), but can work with algebraic expressions that have been grouped via parentheses.

$$\frac{x^2 + 3x + 2}{x + 2} = \frac{(x+1)(x+2)}{x+2} = (x+1)\frac{(x+2)}{(x+2)}$$

$$= (x+1) \bullet 1$$

$$= (x+1)$$

Addition

Adding fractions is more complicated than multiplying them. The key to doing so is understanding that a fraction is a part of a whole. In order for it to make sense to add two fractions, they should be part of the same whole. This means we need a common denominator.

To get a common denominator, we multiply by 1. As 1 is the multiplicitive identity, multiplying by it does not change the value of the original fractions, only their forms. The form of our 1 will depend on the fractions involved.
(It is often easiest to use the least common denominator as it will decrease the likelihood of needing to simplify later)

$$\frac{4}{5}+\frac{2}{3}$$

we will need to express both fractions as part of the same whole. Here, 15 will be that whole.

$$=\frac{4}{5}\bullet\frac{3}{3}+\frac{2}{3}\bullet\frac{5}{5}$$

each term is multiplied by the 1 that will make the denominators equal.

$$=\frac{12}{15}+\frac{10}{15}$$

we chose to do the multiplication first instead of the division

$$=\frac{22}{15}$$

Now that each fraction is part of the same whole, we can add the numerators together to tell the total parts of the whole

It is easier to leave this as an improper fraction, though sometimes we will be able to simplify the sum.

This can also be done with algebraic expressions

$$\frac{x}{3}+\frac{4}{x} \quad =\frac{x}{3}\bullet\frac{x}{x}+\frac{4}{x}\bullet\frac{3}{3}$$

$$=\frac{x^2}{3x}+\frac{12}{3x}$$

$$=\frac{x^2+12}{3x}$$

note: These x's cannot be simplified (divided to 1) because addition is present

Division - also known as Complex Fractions

Division problems involving fractions can be difficult. One of the main reasons is because we need to be sure of where the fraction is (numerator or denominator). Either way, the process is the same.

We want to eliminate the extra division. We do this using multiplication by 1 again.

$$\frac{\frac{5}{3}}{4} = \frac{\frac{5}{3}}{4} \bullet \frac{3}{3}$$

here the extra division is in the numerator. That means we need to eliminate division by 3. We do that by multiplying by 1 in the form of 3/3

$$= \frac{5 \bullet \frac{3}{3}}{4 \bullet 3}$$

we associate the multiplication and division by 3 in the numerator

$$= \frac{5 \bullet 1}{4 \bullet 3}$$

This gives us the multiplicitive identity and eliminates the complex fraction

$$= \frac{5}{12}$$

$$\frac{5}{\frac{3}{4}} = \frac{5}{\frac{3}{4}} \bullet \frac{4}{4}$$

here the extra division is in the denominator. That means we need to eliminate division by 4. We do that by multiplying by 1 in the form of 4/4

$$= \frac{5 \bullet 4}{3 \bullet \frac{4}{4}}$$

we associate the multiplication and division by 4 in the denominator

$$= \frac{5 \bullet 4}{3 \bullet 1}$$

This gives us the multiplicitive identity and eliminates the complex fraction

$$= \frac{20}{3}$$

Note that even though these two problems looked very much alike, they produced vastly different answers because of where the second fraction was located

Lesson 31: Fraction Review

You may want to do work on separate pages.

Evaluate each expression.

1) $-\frac{11}{7} - \frac{1}{2}$

2) $\frac{8}{7} - -\frac{3}{2}$

3) $\frac{5}{3} + -\frac{3}{2}$

4) $1 + -\frac{7}{6}$

5) $\frac{1}{2} - \frac{3}{7}$

6) $\frac{5}{3} - \frac{1}{5}$

7) $-\frac{9}{8} - -\frac{6}{7}$

8) $\frac{1}{3} + -\frac{3}{5}$

Find each product.

9) $-2 \cdot -\frac{9}{10}$

10) $-\frac{11}{6} \cdot -\frac{10}{7}$

11) $-2 \cdot -\frac{5}{6}$

12) $-\frac{2}{3} \cdot -\frac{1}{2}$

13) $-4 \cdot -\frac{1}{2}$

14) $-\frac{1}{3} \cdot -\frac{2}{7}$

15) $-\frac{16}{9} \cdot -\frac{3}{2} \cdot \frac{1}{10}$

16) $2 \cdot 2 \cdot -\frac{11}{8}$

Evaluate each expression.

17) $\dfrac{5}{2 - \frac{7}{5}}$

18) $\left(\frac{1}{2} + \frac{1}{3}\right)^2$

19) $\dfrac{1}{\frac{6}{5} - \frac{1}{3}}$

20) $\frac{3}{2} - 2 \cdot \frac{2}{3}$

21) $\frac{5}{3} + \frac{7}{4} - 2$

22) $\frac{1}{5}\left(\frac{3}{2} + 6\right)$

Simplify each.

23) $\frac{32}{20}$

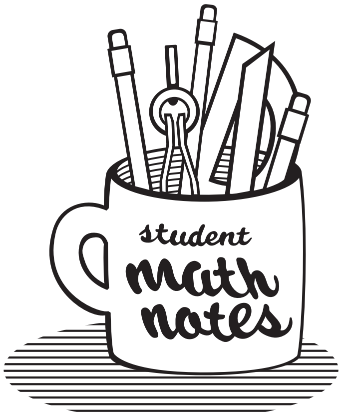
24) $\frac{48}{36}$

25) $\frac{32}{12}$

26) $\frac{27}{18}$

27) $\frac{24}{16}$

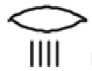
28) $\frac{20}{12}$



Egyptian Fractions

Fractions have a long history of use in mathematics, but they have not always been written as we see them today. About 5,000 years ago the ancient Egyptians represented fractions using symbols like these: 

The  shape means part, and the marks indicate the parts of the whole, so  represents $\frac{1}{3}$.

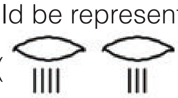
1. What fraction do you think that  represents?

2. Using this pattern, how would you represent $\frac{1}{5}$?

For the most part, the Egyptians used only *unit fractions* (a fraction with the number 1 as the numerator). One of the few fractions that existed in a form other than a unit fraction was $\frac{2}{3}$. It was represented as follows: 
Furthermore, fractions were represented ***without repeating the same*** fractions by using sums of progressively smaller fractions. For example, $\frac{5}{9}$ would not be represented as $\frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$ but rather as $\frac{1}{2} + \frac{1}{18}$. We call this representation an *Egyptian fraction*.

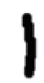
3. What sum of unit fractions could be used to represent $\frac{5}{8}$?

The symbol for $\frac{1}{2}$ is different than the other Egyptian symbols. It is written like this: 
The symbols were written next to each other to show a sum of fractions. For example, $\frac{3}{4}$ would be represented as the sum of $\frac{1}{2}$ and $\frac{1}{4}$. It would be written like this:

4. What fraction could be represented by the two-unit fraction symbols () at the start of this activity?

5. How could you represent $\frac{5}{8}$ using Egyptian symbols?

The Egyptians used different hieroglyphs to represent different values:

1	10	100	1000
			
Single Stroke	Cattle Hobble	Rope Coil	Lotus Plant

They also used these symbols to represent fractions. For example, according to the table above, we can represent $\frac{1}{23}$ in the following manner: 

6. How would you represent $\frac{1}{12}$ using Egyptian hieroglyphs?

7. How would you represent $\frac{1}{1,234}$ using Egyptian hieroglyphs?

A Method to Determine Egyptian Fractions

A useful method for translating a given fraction into an Egyptian fraction is to determine if $\frac{2}{3}$ is less than the given fraction. If it is not, determine the largest unit fraction that is less than the given fraction. List the largest unit fraction (in some cases $\frac{2}{3}$) as the first part of the Egyptian fraction. Then subtract it from the given fraction. Use the result, or the "leftover" fraction, to find the next unit fraction that is smaller than the "leftover" fraction. Repeat this process until the "leftover" fraction is a unit fraction.

For example, in determining the Egyptian fraction for $\frac{5}{6}$, first determine if $\frac{2}{3}$ is less than $\frac{5}{6}$. Since $\frac{2}{3}$ is equivalent to $\frac{4}{6}$, and $\frac{4}{6}$ is less than $\frac{5}{6}$, we know that $\frac{2}{3}$ can be used as the first part of the Egyptian fraction for $\frac{5}{6}$. So, we subtract $\frac{2}{3}$ from $\frac{5}{6}$.
We know that $\frac{5}{6} - \frac{2}{3} = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$. Since $\frac{1}{6}$ is in the form of a unit fraction, we use the "leftover" of $\frac{1}{6}$ to complete the Egyptian fraction for $\frac{5}{6}$. Therefore, an Egyptian fraction for $\frac{5}{6}$ is $\frac{2}{3} + \frac{1}{6}$. Let's start this process using the fraction $\frac{3}{8}$.

8. Is $\frac{3}{8}$ greater than or less than $\frac{1}{2}$? How do you know?

It is helpful to be able to compare fractions to $\frac{1}{2}$ without finding common denominators. We can compare the numerator of a fraction to its denominator in the following ways:

- If the numerator is less than $\frac{1}{2}$ the denominator, then the fraction is less than $\frac{1}{2}$;
- If the numerator is greater than $\frac{1}{2}$ the denominator, then the fraction is greater than $\frac{1}{2}$.

This is also an efficient strategy for comparing two fractions when one is greater than $\frac{1}{2}$ and the other is less than $\frac{1}{2}$. Finding common denominators becomes unnecessary. Consider $\frac{9}{14}$ and $\frac{5}{12}$. We know that $\frac{9}{14}$ is greater than $\frac{1}{2}$ because 9 is greater than 7 (which is $\frac{1}{2}$ of 14). We also know that $\frac{5}{12}$ is less than $\frac{1}{2}$ because 5 is less than 6 (which is $\frac{1}{2}$ of 12). Therefore, by using the benchmark fraction of $\frac{1}{2}$, we know that $\frac{9}{14} > \frac{5}{12}$.

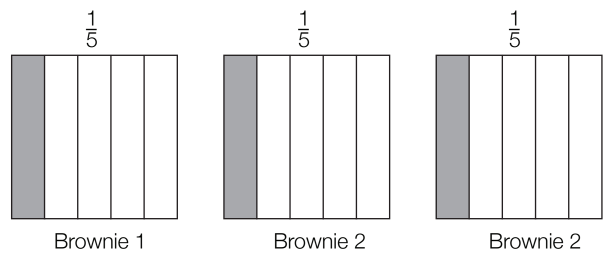
9. Using the benchmark of $\frac{1}{2}$ to compare $\frac{3}{7}$ and $\frac{5}{9}$, determine which fraction is greater. Explain your reasoning.

Let's return to finding an Egyptian fraction to represent $\frac{3}{8}$. We saw that $\frac{3}{8}$ is less than $\frac{1}{2}$. A unit fraction that is less than $\frac{3}{8}$ is $\frac{1}{4}$. List $\frac{1}{4}$ as the first unit fraction in the Egyptian fraction for $\frac{3}{8}$ and then subtract $\frac{1}{4}$ from $\frac{3}{8}$, leaving 1/8. This "leftover" portion of $\frac{3}{8}$ is already in the form of a unit fraction, so the process is complete. An Egyptian fraction for $\frac{3}{8}$ is $\frac{1}{4} + \frac{1}{8}$, or these symbols:. 

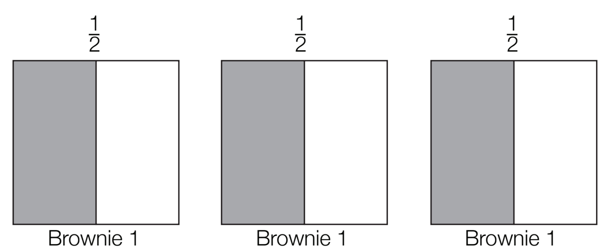
10. Determine an Egyptian fraction for $\frac{5}{9}$.

11. Determine two different ways to represent $\frac{3}{4}$ using Egyptian fractions. (Hint: You are not required to use the largest unit fraction less than $\frac{3}{4}$.)

This method of determining Egyptian fractions has a useful application to sharing situations. Consider sharing 3 brownies among 5 people. We could share the brownies fairly by dividing each one into 5 pieces and giving each person 1 piece from each brownie. Each person receives $\frac{3}{5}$ of a brownie, as illustrated below by shaded portions.



Now let's apply the method for determining Egyptian fractions. If we begin with the same 3 brownies and find the largest unit fraction of a brownie that we can give to each of the 5 people, we could give each person $\frac{1}{2}$ brownie and still have $\frac{1}{2}$ brownie left to share.



12. Divide the last $\frac{1}{2}$ brownie into 5 equal pieces and share them evenly among 5 people. What fraction of a brownie will each person receive? How do you know?

13. Using this process as a guide, determine an Egyptian fraction representation for $\frac{3}{5}$.

14. Explain how the method for determining Egyptian fractions could be used to share 4 pizzas fairly among 5 people. Represent your solution and strategy with both a drawing and a written description.

In the brownie example, we saw that $\frac{1}{2}$ brownie results in a bigger piece than $\frac{1}{5}$ brownie. We can compare these fractions without finding common denominators. This is true for all unit fractions.

15. Explain how you know that $\frac{1}{2}$ brownie is greater than $\frac{1}{5}$ brownie. Describe how the strategy for comparing fractions can be applied to all unit fractions.

16. Explain how to use this strategy to compare $\frac{4}{9}$ and $\frac{4}{11}$.

17. Now imagine 2 pizzas of the same size. One has $\frac{7}{8}$ remaining, and the other has $\frac{11}{12}$ remaining. How can this fraction comparison strategy help us to determine which pizza has more remaining?

18. Use this strategy to compare $\frac{11}{13}$ and $\frac{17}{19}$. Explain your reasoning.

We can use Egyptian fractions to compare fractions. For example, consider $\frac{5}{8}$ and $\frac{6}{10}$. Both fractions are greater than $\frac{1}{2}$, so the benchmark of $\frac{1}{2}$ strategy is not sufficient for determining which fraction is greater. The numerators are not the same, and the same number of pieces are not "missing" from the fractions, so neither of these strategies will work on their own, either. Let's see how finding the Egyptian fraction representation might help.

With Egyptian fractions: $\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$, and $\frac{6}{10} = \frac{1}{2} + \frac{1}{10}$. Both fractions are greater than $\frac{1}{2}$, so we must compare the parts that are greater than $\frac{1}{2}$. Since $\frac{1}{8}$ is greater than $\frac{1}{10}$, we can conclude that $\frac{5}{8}$ is greater than $\frac{6}{10}$. We just used a combination of strategies to determine which fraction is greater.

19. Use Egyptian fractions to compare $\frac{8}{15}$ and $\frac{6}{11}$. Explain your reasoning.

20. When ordering a list of fractions, it is often helpful to apply several comparison strategies. Order the following list of fractions from least to greatest:

$\frac{11}{13}$ $\frac{3}{5}$ $\frac{3}{7}$ $\frac{8}{14}$ $\frac{9}{16}$ $\frac{8}{10}$.

Describe the strategies you use to complete this task.

Can You …

- compare $\frac{22}{23}$ and $\frac{26}{27}$?
- determine the unit fraction sum for $\frac{2}{29}$?
- find three different unit fraction sums for $\frac{7}{8}$?

Did You Know That …

- Fibonacci proved that every simple fraction can be represented as the sum of unit fractions? The method for finding the sum is called a *greedy algorithm*.
- an infinite number of unit fraction sum representations exist for every simple fraction?

Mathematical Content

Representing fractions, comparing and ordering fractions, adding and subtracting fractions

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